Matrices and Determinants

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1. If $f = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix}$ and $g = (x-y)(y-z)(z-x)$, then $\frac{f}{g}$ is:

(A) $xy + yz + zx$ **(B)** $x^2 + y^2 + z^2$

(C) $x^2 + y^2 + z^2 - xy - yz - zx$ **(D)** none of above

2. If $g(x) = \begin{vmatrix} a^{-x} & e^{x \log_e a} & x^2 \\ a^{-3x} & e^{3x \log_e a} & x^4 \\ a^{-5x} & e^{5x \log_e a} & 1 \end{vmatrix}$, then:

(A) $g(x) + g(-x) = 0$ **(B)** $g(x) - g(-x) = 0$

(C) $g(x) \times g(-x) = 0$ **(D)** none of these

3. If $x \neq 0, y \neq 0, z \neq 0$ and $\begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+z & 1+z & 1+3z \end{vmatrix} = 0$, then $x^{-1} + y^{-1} + z^{-1}$ is equal to :

(A) -1 **(B)** -2 **(C)** -3 **(D)** $\frac{1}{3}$

4. If $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$ then $A^3 - rA^2 - qA =$

(A) pI **(B)** qI

(C) rI **(D)** none of these, I is third order unit matrix

***5.** If $2x - y = \begin{bmatrix} 3 & -3 & 0 \\ 3 & 3 & 2 \end{bmatrix}$ and $2y + x = \begin{bmatrix} 4 & 1 & 5 \\ -1 & 4 & -4 \end{bmatrix}$, then:

(A) $x + y = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & -2 \end{bmatrix}$ **(B)** $x = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$

(C) $x - y = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 2 \end{bmatrix}$ **(D)** $y = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 1 & -2 \end{bmatrix}$

6. If $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then $f(x+y)$ is equal to:

(A) $f(x) + f(y)$ **(B)** $f(x) - f(y)$ **(C)** $f(x) \cdot f(y)$ **(D)** None of these

7. If $AB = 0$ where $A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and $B = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ then $|\theta - \phi|$ is equal to:

(A) 0 **(B)** $\frac{\pi}{2}$ **(C)** $\frac{\pi}{4}$ **(D)** π

8. If $A=\begin{bmatrix}1 & \omega & \omega^2\\ \omega & \omega^2 & 1\\ \omega^2 & 1 & \omega\end{bmatrix}, B=\begin{bmatrix}\omega & \omega^2 & 1\\ \omega^2 & 1 & \omega\\ \omega & \omega^2 & 1\end{bmatrix}$ and $C=\begin{bmatrix}1\\ \omega\\ \omega^2\end{bmatrix}$ where ω is the complex cube root of 1 then $(A+B)C$ is equal to:

(A) $\begin{bmatrix}0\\0\\0\end{bmatrix}$ **(B)** $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ **(C)** $\begin{bmatrix}1\\0\\1\end{bmatrix}$ **(D)** $\begin{bmatrix}1\\1\\1\end{bmatrix}$

9. If $A=\begin{bmatrix}0 & c & -b\\ -c & 0 & a\\ b & -a & 0\end{bmatrix}$ and $B=\begin{bmatrix}a^2 & ab & ac\\ ba & b^2 & bc\\ ca & cb & c^2\end{bmatrix}$ then AB is equal to:

(A) O **(B)** I **(C)** 2I **(D)** None of these

10. If $\begin{bmatrix}x+y & y\\ 2x & x-y\end{bmatrix}\begin{bmatrix}2\\ -1\end{bmatrix}=\begin{bmatrix}3\\ 2\end{bmatrix}$ then $x\cdot y$ is equal to :

(A) -5 **(B)** 5 **(C)** 4 **(D)** 6

11. If $a^{-1}+b^{-1}+c^{-1}=0$ such that $\begin{vmatrix}1+a & 1 & 1\\ 1 & 1+b & 1\\ 1 & 1 & 1+c\end{vmatrix}=\lambda$ then the value of λ is :

(A) 0 **(B)** abc **(C)** $-abc$ **(D)** None of these

12. The value of 'λ' if $ax^4+bx^3+cx^2+50x+d=\begin{vmatrix}x^3-14x^2 & -x & 3x+\lambda\\ 4x+1 & 3x & x-4\\ -3 & 4 & 0\end{vmatrix}$, is :

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

13. The values of θ lying between $\theta=0$ and $\theta=\pi/2$ and satisfying the equation

$\begin{vmatrix}1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta\\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta\\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta\end{vmatrix}=0$ are given by :

(A) $\pi/24, 5\pi/24$ **(B)** $7\pi/24, 11\pi/24$ **(C)** $5\pi/24, 7\pi/24$ **(D)** $11\pi/24, \pi/24$

14. If $f(x)=\begin{vmatrix}1-x & -1 & 0\\ 2 & 3-x & 1\\ 4 & -2 & 5-x\end{vmatrix}$, number of real roots of $f(x)=0$ is ________.

(A) 1 **(B)** 0 **(C)** 2 **(D)** 3

15. If $A=\begin{bmatrix}\cos^2\theta & \sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$, $B=\begin{bmatrix}\cos^2\phi & \sin\phi\cos\phi\\ \sin\phi\cos\phi & \sin^2\phi\end{bmatrix}$ and θ and ϕ differs by $\frac{\pi}{2}$, then $AB=$

(A) I **(B)** O **(C)** $-I$ **(D)** None of these

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16. $\begin{vmatrix} 1+i & 1-i & i \\ 1-i & i & 1+i \\ i & 1+i & 1-i \end{vmatrix} =$

(A) $-4-7i$ **(B)** $4+7i$ **(C)** $3+7i$ **(D)** $7+4i$

17. If ω is a cube root of unity, then $\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} =$

(A) x^3+1 **(B)** $x^3+\omega$ **(C)** $x^3+\omega^2$ **(D)** x^3

18. If $\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix} = k(x+y+z)(x-z)^2$, then $k =$

(A) $2xyz$ **(B)** 1 **(C)** xyz **(D)** $x^2y^2z^2$

19. If – 9 is a root of the equation $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$ then the other two roots are :

(A) 2, 7 **(B)** – 2, 7 **(C)** 2, –7 **(D)** – 2, –7

20. If $A = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}, B = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}, C = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$, then which relation is correct :

(A) $A = B$ **(B)** $A = C$ **(C)** $B = C$ **(D)** None of these

21. $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} =$

(A) $a^3+b^3+c^3-3abc$ **(B)** $3abc-a^3-b^3-c^3$

(C) $a^3+b^3+c^3-a^2b-b^2c-c^2a$ **(D)** $(a+b+c)(a^2+b^2+c^2+ab+bc+ca)$

22. If ω is a cube root of unity and $\Delta = \begin{vmatrix} 1 & 2\omega \\ \omega & \omega^2 \end{vmatrix}$, then Δ^2 is equal to :

(A) $-\omega$ **(B)** ω **(C)** 1 **(D)** ω^2

23. If $\Delta_1 = \begin{vmatrix} 1 & 0 \\ a & b \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 1 & 0 \\ c & d \end{vmatrix}$, then $\Delta_2\Delta_1$ is equal to :

(A) ac **(B)** bd **(C)** $(b-a)(d-c)$ **(D)** None of these

24. For the matrix $A=\begin{bmatrix}1 & 1 & 0\\ 1 & 2 & 1\\ 2 & 1 & 0\end{bmatrix}$, which of the following is correct:

(A) $A^3+3A^2-I=O$ **(B)** $A^3-3A^2-I=O$

(C) $A^3+2A^2-I=O$ **(D)** $A^3-A^2+I=O$

25. If $A=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$, $B=\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}$ and $(A+B)^2=A^2+B^2$, then the value of a and b are:

(A) $a=4, b=1$ **(B)** $a=1, b=4$ **(C)** $a=0, b=4$ **(D)** $a=2, b=4$

26. If $A=\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}$, then $A^2-5A=$

(A) I **(B)** $14\,I$ **(C)** 0 **(D)** None of these

27. If matrix $A=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$, then $A^{16}=$

(A) $\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$ **(B)** $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ **(C)** $\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}$ **(D)** $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

28. If $A=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$, then $A^{100}=$

(A) $2^{100}A$ **(B)** $2^{99}A$ **(C)** $2^{101}A$ **(D)** None of these

29. If $A=\begin{bmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}$, then $A^n=$

(A) $\begin{bmatrix}na & 0 & 0\\ 0 & nb & 0\\ 0 & 0 & nc\end{bmatrix}$ **(B)** $\begin{bmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}$ **(C)** $\begin{bmatrix}a^n & 0 & 0\\ 0 & b^n & 0\\ 0 & 0 & c^n\end{bmatrix}$ **(D)** None of these

30. The matrix $\begin{bmatrix}2 & \lambda & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{bmatrix}$ is non singular, if :

(A) $\lambda \neq -2$ **(B)** $\lambda \neq 2$ **(C)** $\lambda \neq 3$ **(D)** $\lambda \neq -3$

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31. $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = Ax+B$. Then $A+2B$ is equal to:

(A) 0 **(B)** 4 **(C)** 1 **(D)** 2

32. If $f(x)g(x)$ and $h(x)$ are three polynomials of degree 3 then $\phi(x) = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \\ f'''(x) & g'''(x) & h'''(x) \end{vmatrix}$ is a polynomial of degree :

(A) 3 **(B)** 4 **(C)** 5 **(D)** None of these

33. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \ge 1$.

(A) $A^n = 2^{n-1}A + (n-1)I$ **(B)** $A^n = nA + (n-1)I$

(C) $A^n = 2^{n-1}A - (n-1)I$ **(D)** $A^n = nA - (n-1)I$

34. A square matrix P satisfies $P^2 = I - P$, where I is the identity matrix. If $P^n = 5I - 8P$, then n is equal to:

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7

35. If $3A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$ and $AA^T = I$. Then $x+y$ is equal to :

(A) -3 **(B)** -2 **(C)** -1 **(D)** 0

***36.** Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then

(A) $A^2 - 4A - 5I_3 = 0$ **(B)** $A^{-1} = \frac{1}{5}(A - 4I_3)$

(C) A^3 is not invertible **(D)** A^2 is invertible

37. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P^T\left(Q^{2005}\right)P$ is equal to :

(A) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$ **(B)** $\begin{bmatrix} \frac{\sqrt{3}}{2} & 2005 \\ 1 & 0 \end{bmatrix}$ **(C)** $\begin{bmatrix} 1 & 2005 \\ \frac{\sqrt{3}}{2} & 1 \end{bmatrix}$ **(D)** $\begin{bmatrix} 1 & \frac{\sqrt{3}}{2} \\ 0 & 2005 \end{bmatrix}$

***38.** If $A+B+C = \pi, e^{i\theta} = \cos\theta + i\sin\theta$ and $z = \begin{vmatrix} e^{2iA} & e^{-iC} & e^{-iB} \\ e^{-iC} & e^{2iB} & e^{-iA} \\ e^{-iB} & e^{-iA} & e^{-2iC} \end{vmatrix}$, then :

(A) $\text{Re}(z) = 4$ **(B)** $\text{Im}(z) = 0$ **(C)** $\text{Re}(z) = -4$ **(D)** $\text{Im}(z) = 1$

39. The value of $\begin{vmatrix} 1 & 1 & 1 \\ \left(2^x+2^{-x}\right)^2 & \left(3^x+3^{-x}\right)^2 & \left(5^x+5^{-x}\right)^2 \\ \left(2^x-2^{-x}\right)^2 & \left(3^x-3^{-x}\right)^2 & \left(5^x-5^{-x}\right)^2 \end{vmatrix}$ is :

(A) 0 **(B)** 30^x **(C)** 30^{-x} **(D)** None of these

40. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}, B = (adj\ A)$ and $C = 5A$, then $\dfrac{|adj\ B|}{|C|}$ is equal to :

(A) 5 **(B)** 25 **(C)** -1 **(D)** 1

41. If a, b, c, are in A.P. and $f(x) = \begin{vmatrix} x+a & x^2+1 & 1 \\ x+b & 2x^2-1 & 1 \\ x+c & 3x^2-2 & 1 \end{vmatrix}$, then $f'(x)$ is :

(A) 0 **(B)** 1 **(C)** $a+bc$ **(D)** $\dfrac{abc}{a+b+c}$

***42.** The value of x for which $\begin{vmatrix} x & 2 & 2 \\ 3 & x & 2 \\ 3 & 3 & x \end{vmatrix} + \begin{vmatrix} 1-x & 2 & 4 \\ 2 & 4-x & 8 \\ 4 & 8 & 16-x \end{vmatrix} > 33$ is :

(A) $0 < x < 1$ **(B)** $-\dfrac{1}{2} < x < \dfrac{1}{2}$ **(C)** $x < -\dfrac{1}{7}$ **(D)** $x > 1$

***43.** Let $f(n) = \begin{vmatrix} n & n+1 & n+2 \\ {}^nP_n & {}^{n+1}P_{n+1} & {}^{n+2}P_{n+2} \\ {}^nC_n & {}^{n+1}C_{n+1} & {}^{n+2}C_{n+2} \end{vmatrix}$ where the symbols have their usual meanings. Then $f(n)$ is divisible by :

(A) n^2+n+1 **(B)** $(n+1)!$ **(C)** $n!$ **(D)** None of these

44. If $\Delta = \begin{vmatrix} f(x) & f\left(\frac{1}{x}\right)+f(x) \\ 1 & f\left(\frac{1}{x}\right) \end{vmatrix} = 0$ where, $f(x) = a + bx^n$ and $f(2) = 17$, then $f(5)$ is :

(A) 126 **(B)** 326 **(C)** 428 **(D)** 626

45. The value of x, so that $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix} = 0$, is:

(A) $\dfrac{-7 \pm \sqrt{35}}{2}$ **(B)** $\dfrac{-9 \pm \sqrt{53}}{2}$ **(C)** ± 2 **(D)** 0

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Paragraph for Questions 46 to 48

Elementary Transformation of a matrix:

The following operation on a matrix are called elementary operations (transformations)

1. The interchange of any two rows (or columns)

2. The multiplication of the elements of any row (or column) by any nonzero number

3. The addition to the elements of any row (or column) the corresponding elements of any other row (or column) multiplied by any number

Echelon Form of matrix :

A matrix A is said to be in echelon form if

(i) every row of A which has all its elements 0, occurs below row, which has a non-zero elements

(ii) the first non-zero element in each non –zero row is 1.

(iii) The number of zeros before the first non zero elements in a row is less than the number of such zeros in the next now.

[A row of a matrix is said to be a zero row if all its elements are zero]

Note: Rank of a matrix does not change by application of any elementary operations

For example $\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 3 & 6 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ are echelon forms

The number of non-zero rows in the echelon form of a matrix is defined as its RANK.

For example we can reduce the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$ into echelon form using following elementary row transformation.

(i) $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1 \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(ii) $R_2 \to R_2 - 2R_1 \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

This is the echelon form of matrix A

Number of nonzero rows in the echelon form =2 $\Rightarrow$ Rank of the matrix A is 2

46. Rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$ is :

(A) 1 **(B)** 2 **(C)** 3 **(D)** 0

47. Rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 4 & 4 \\ 3 & 4 & 5 & 2 \end{bmatrix}$ is :

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

48. The echelon form of the matrix $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 1 \end{bmatrix}$ is :

(A) $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$ **(C)** $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ **(D)** $\begin{bmatrix} 1 & 3 & 4 & -\frac{3}{2} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

49. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ \lambda & 2 & 4 \\ 2 & -3 & 1 \end{bmatrix}$ is 3 if :

(A) $\lambda \neq \frac{18}{11}$ **(B)** $\lambda = \frac{18}{11}$ **(C)** $\lambda = -\frac{18}{11}$ **(D)** None of these

50. The rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ is equal to :

(A) 1 **(B)** 2 **(C)** 3 **(D)** None of these

51. If 3, – 2 are the Eigen values of a non-singular matrix A and $|A| = 4$, then the Eigen values of $adj(A)$ are :

(A) $\frac{3}{4}, \frac{-1}{2}$ **(B)** $\frac{4}{3}, -2$ **(C)** 12, – 8 **(D)** – 12, 8

52. Let p a non singular matrix $1 + P + P^2 + ... + P^n = O$. ($O$ denotes the null matrix), then $P^{-1} =$

(A) P^n **(B)** $-P^n$ **(C)** $-(1 + P + ... + P^n)$ **(D)** None of these

53. If $P = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ -2 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} -4 & -5 & -6 \\ 0 & 0 & 1 \end{bmatrix}$ then $P_{22} =$

(A) 40 **(B)** – 40 **(C)** – 20 **(D)** 20

54. If $1, \omega, \omega^2$ are the cube roots of unity, then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to :

(A) 0 **(B)** 1 **(C)** ω **(D)** ω^2

55. Let P be a non-singular matrix $I + P + P^2 + ... + P^n = O$ (O denotes the null matrix), then P^{-1} is :

(A) P^n **(B)** $-P^n$

(C) $-(I + P + ... + P^n)$ **(D)** None of these

56. If D = diagonal $\left[d_1, d_2, d_3, ..., d_n\right]$ where $d_i \neq 0 \forall i = 1, 2, 3, ..., n$ then D^{-1} is equal to :

(A) O

(B) I_n

(C) diagonal $\left(d_1^{-1}, d_2^{-1}, ... d_n^{-1}\right)$

(D) None of above

57. Let A be an orthogonal non-singular matrix of order n, then $\left|A - I_n\right|$ is equal to :

(A) $\left|I_n - A\right|$ **(B)** $|A|$ **(C)** $|A|\left|I_n - A\right|$ **(D)** $(-1)^n |A|\left|I_n - A\right|$

58. 'A' is any square matrix, then det $\left|A - A^T\right|$ is equal to :

(A) 0

(B) 1

(C) can be 0 or a perfect square

(D) cannot be determined

59. In a ΔABC, if $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$, then $\sin^2 A + \sin^2 B + \sin^2 C$ is equal to :

(A) $\frac{9}{4}$ **(B)** $\frac{4}{9}$ **(C)** 1 **(D)** $3\sqrt{3}$

60. If a, b, c are the sides of a ΔABC opposite angle A, B, C respectively, then

$$\Delta = \begin{vmatrix} a^2 & b\sin A & c\sin A \\ b\sin A & 1 & \cos(B-C) \\ c\sin A & \cos(B-C) & 1 \end{vmatrix}$$ equals :

(A) $\sin A - \sin C \sin B$

(B) abc

(C) 1

(D) 0

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Paragraph for Questions 61 to 63

Consider the determinant, $\Delta = \begin{vmatrix} p & q & r \\ x & y & z \\ l & m & n \end{vmatrix}$.

M_{ij} denotes the minor of an element in i^{th} row, and j^{th} column

C_{ij} denotes the cofactor of an element in i^{th} row and j^{th} column

61. The value of $p \cdot C_{21} + q \cdot C_{22} + r \cdot C_{23}$ is :

(A) 0 **(B)** $-\Delta$ **(C)** Δ **(D)** Δ^2

62. The value of $x \cdot C_{21} + y \cdot C_{22} + z \cdot C_{23}$ is :

(A) 0 **(B)** $-\Delta$ **(C)** Δ **(D)** Δ^2

63. The value of $q \cdot M_{12} - y \cdot M_{22} + m \cdot M_{32}$ is :

(A) 0 **(B)** $-\Delta$ **(C)** Δ **(D)** Δ^2

64. A and B are square matrices and A is non-singular matrix, $\left(A^{-1}BA\right)^n, n \in I^+$, is equal to :

(A) $A^{-n}B^nA^n$ **(B)** $A^nB^nA^{-n}$ **(C)** $A^{-1}B^nA$ **(D)** $A^{-n}BA^n$

65. The value of a, b, c when $\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal, are :

(A) $\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{6}}, \pm\frac{1}{\sqrt{2}}$ **(B)** $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{6}}$ **(C)** $\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{6}} \pm\frac{1}{\sqrt{3}}$ **(D)** $\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}} \pm\frac{1}{\sqrt{2}}$

66. The equations $2x + y = 5, x + 3y = 5, x - 2y = 0$ have:

(A) no solution **(B)** one solution **(C)** two solutions **(D)** infinitely many solutions

Passage for Question 67 to 70

Consider a system of linear equation in three variables x, y, z

$$a_1x + b_1y + c_1z = d_1; \ a_2x + b_2y + c_2z = d_2; \ a_3x + b_3y + c_3z = d_3$$

The system can be expressed by matrix equation $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

- if A is non-singular matrix then the solution of above system can be found by $X = A^{-1}B$, the solution in this case is unique.
- if A is a singular matrix i.e. $|A| = 0$, then the system will have
- no unique solution if $(Adj\,A)B = 0$
- no solution (i.e. it is inconsistent) if $(Adj\,A)B \neq 0$

Where Adj A is the adjoint of the matrix A, which is obtained by taking transpose of the matrix obtained by replacing each element of matrix A with corresponding cofactors.

Now consider the following matrix.

$$A = \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}, B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}, U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}, V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

67. The system $AX = U$ has infinitely many solution if:

(A) $c = d, ab = 1$ **(B)** $c = d, h = g$ **(C)** $ab = 1, h = g$ **(D)** $c = d, h = g, ab = 1$

68. If $AX = U$ has infinitely many solutions then the equation $BX = V$ has:

(A) unique solution **(B)** infinitely many solution
(C) no solution **(D)** either infinitely many solutions or no solution

***69.** If $AX = U$ has infinitely many solutions then the equation $BX = V$ is consistent if

(A) $a = 0$ **(B)** $d = 0$ **(C)** $f = 0$ **(D)** $adf \neq 0$

***70.** Consider the following statements:

A: if $AX = U$; has infinite solutions and $cf \neq 0$, then one solution of $BX = V$ is $(0,0,0)$

R: if a system has infinite solutions then one solution must be trivial. Then

(A) A and R are both correct and R is correct explanation of A
(B) A and R both are correct but R is not correct explanation of A
(C) A is correct R is wrong
(D) A and R are both wrong

71. If t is real and $\lambda = \dfrac{t^2 - 3t + 4}{t^2 + 3t + 4}$, then the number of solutions of the system of equations $3x - y + 4z = 3$, $x + 2y - 3z = -2$, $6x + 5y + \lambda z = -3$ is :

(A) one **(B)** Two **(C)** zero **(D)** infinite

72. The set of equations $\lambda x - y + (\cos\theta)z = 0$; $3x + y + 2z = 0$; $(\cos\theta)x + y + 2z = 0$; $0 \le \theta < 2\pi$, has non–trivial solutions.

(A) for no value of λ and θ **(B)** for all value of λ and θ
(C) for all values of λ and only two value of θ
(D) For only one value of λ and all values of θ

***73.** Let $\Delta(x) = \begin{vmatrix} x+a & x+b & x+a-c \\ x+b & x+c & x-1 \\ x+c & x+d & x-b+d \end{vmatrix}$ and $\int_0^2 \Delta(x)\,dx = -16$ where a, b, c, d are in AP, then the common difference of the AP is:

(A) 1 **(B)** 2 **(C)** -2 **(D)** None of these

***74.** Let $\{\Delta_1, \Delta_2, \Delta_3, \ldots, \Delta_k\}$ be the set of third order determinants that can be made with the distinct nonzero real numbers $a_1, a_2, a_3, \ldots, a_9$. Then:

(A) $k = 9!$ **(B)** $\sum_{i=1}^{k} \Delta_i = 0$
(C) at least one $\Delta_i = 0$ **(D)** None of these

75. If $a \neq p, b \neq q, c \neq r$ and the system of equations $px + by + cz = 0$, $ax + qy + cz = 0$, $ax + by + rz = 0$ has a non-zero solution, then value of $\dfrac{p}{p-a} + \dfrac{q}{q-b} + \dfrac{r}{r-c}$ is:

(A) -1 **(B)** -2 **(C)** 1 **(D)** 2

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76. If $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$ are the given determinants, then :

(A) $\Delta_1 = 3(\Delta_2)^2$ **(B)** $\frac{d}{dx}(\Delta_1) = 3\Delta_2$ **(C)** $\frac{d}{dx}(\Delta_1) = 2(\Delta_2)^2$ **(D)** $\Delta_1 = 3\Delta_2^{3/2}$

77. In the determinant $\begin{vmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{vmatrix}$, the ratio of the co-factor to its minor of the element –3 is :

(A) –1 **(B)** 0 **(C)** 1 **(D)** 2

78. If value of a third order determinant is 11, then the value of the square of the determinant formed by the cofactors will be :

(A) 11 **(B)** 121 **(C)** 1331 **(D)** 14641

79. Consider the system of linear equations $a_1x + b_1y + c_1z + d_1 = 0$, $a_2x + b_2y + c_2z + d_2 = 0$ and $a_3x + b_3y + c_3z + d_3 = 0$. Let us denote by $\Delta(a,b,c)$ the determinant $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$. If $\Delta(a,b,c) \neq 0$, then the value of x in the unique solution of the above equations is :

(A) $\frac{\Delta(bcd)}{\Delta(abc)}$ **(B)** $\frac{-\Delta(bcd)}{\Delta(abc)}$ **(C)** $\frac{\Delta(acd)}{\Delta(abc)}$ **(D)** $-\frac{\Delta(abd)}{\Delta(abc)}$

80. The value of the determinant $\begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$ is :

(A) $2(10!\,11!)$ **(B)** $2(10!\,13!)$ **(C)** $2(10!\,11!\,12!)$ **(D)** $2(11!\,12!\,13!)$

81. The cofactor of the element '4' in the determinant $\begin{vmatrix} 1 & 3 & 5 & 1 \\ 2 & 3 & 4 & 2 \\ 8 & 0 & 1 & 1 \\ 0 & 2 & 1 & 1 \end{vmatrix}$ is :

(A) 4 **(B)** 10 **(C)** – 10 **(D)** – 4

82. If $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and A_1, B_1, C_1 denote the co-factors of a_1, b_1, c_1 respectively, then the value of the determinant $\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$ is :

(A) Δ **(B)** Δ^2 **(C)** Δ^3 **(D)** 0

83. If in the determinant $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, A_1, B_1, C_1 etc. be the co-factors of a_1, b_1, c_1 etc., then which of the following relations is incorrect :

(A) $a_1A_1 + b_1B_1 + c_1C_1 = \Delta$ **(B)** $a_2A_2 + b_2B_2 + c_2C_2 = \Delta$

(C) $a_3A_3 + b_3B_3 + c_3C_3 = \Delta$ **(D)** $a_1A_2 + b_1B_2 + c_1C_2 = \Delta$

84. If A_1, B_1, C_1 .... are respectively the co-factors of the elements a_1, b_1, c_1 ,...... of the determinant $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, then $\begin{vmatrix} B_2 & C_2 \\ B_3 & C_3 \end{vmatrix} =$

(A) $a_1\Delta$ **(B)** $a_1a_3\Delta$ **(C)** $(a_1 + b_1)\Delta$ **(D)** None of these

85. Let $A = [a_{ij}]_{n\times n}$ be a square matrix and let c_{ij} be cofactor of a_{ij} in A. If $C = [c_{ij}]$, then :

(A) $|C| = |A|$ **(B)** $|C| = |A|^{n-1}$ **(C)** $|C| = |A|^{n-2}$ **(D)** None of these

86. $x + ky - z = 0, 3x - ky - z = 0$ and $x - 3y + z = 0$ has non-zero solution for $k =$

(A) – 1 **(B)** 0 **(C)** 1 **(D)** 2

87. The number of solutions of equations $x + y - z = 0$, $3x - y - z = 0, x - 3y + z = 0$ is :

(A) 0 **(B)** 1 **(C)** 2 **(D)** Infinite

88. If $x + y - z = 0,\ 3x - \alpha y - 3z = 0,\ x - 3y + z = 0$ has non zero solution, then $\alpha =$

(A) – 1 **(B)** 0 **(C)** 1 **(D)** – 3

89. If $\Delta(x) = \begin{vmatrix} x^n & \sin x & \cos x \\ n! & \sin\frac{n\pi}{2} & \cos\frac{n\pi}{2} \\ a & a^2 & a^3 \end{vmatrix}$, then the value of $\frac{d^n}{dx^n}[\Delta(x)]$ at $x = 0$ is :

(A) – 1 **(B)** 0 **(C)** 1 **(D)** Dependent of a

90. The inverse of $\begin{bmatrix} 3 & 5 & 7 \\ 2 & -3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ is :

(A) $\begin{bmatrix} 7 & 3 & -26 \\ 3 & 1 & -11 \\ -5 & -2 & 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} 7 & 3 & -26 \\ 3 & 1 & 11 \\ -5 & -2 & 1 \end{bmatrix}$ **(C)** $\begin{bmatrix} 3 & 1 & 11 \\ 7 & 3 & -26 \\ -5 & 2 & 1 \end{bmatrix}$ **(D)** None of these

Matrices and Determinants

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***91.** Let A and B be two nonsingular square matrices, A^T and B^T are the transpose matrices of A and B, respectively, then which of the following are correct?

(A) $B^T AB$ is symmetric matrix if A is symmetric

(B) $B^T AB$ is symmetric matrix if B is symmetric

(C) $B^T AB$ is skew-symmetric matrix for every matrix A

(D) $B^T AB$ is skew-symmetric matrix if A is skew-symmetric

***92.** If $A(\theta) = \begin{bmatrix} \sin\theta & i\cos\theta \\ i\cos\theta & \sin\theta \end{bmatrix}$, then which of the following is not true?

(A) $A(\theta)^{-1} = A(\pi - \theta)$ **(B)** $A(\theta) + A(\pi + \theta)$ is a null matrix

(C) $A(\theta)$ is invertible for all $\theta \in R$ **(D)** $A(\theta)^{-1} = A(-\theta)$

***93.** If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then :

(A) $adj(adj)A = A$ **(B)** $|adj(adj\ A)| = 1$ **(C)** $|adj\ A| = 1$ **(D)** None of these

***94.** If $A^{-1} = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & -1/3 \end{bmatrix}$, then:

(A) $|A| = -1$ **(B)** $\text{adj}A = \begin{bmatrix} -1 & 1 & -2 \\ 0 & -3 & -1 \\ 0 & 0 & 1/3 \end{bmatrix}$

(C) $A = \begin{bmatrix} 1 & 1/3 & 7 \\ 0 & 1/3 & 1 \\ 0 & 0 & -3 \end{bmatrix}$ **(D)** $A = \begin{bmatrix} 1 & -1/3 & -7 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

***95.** Which of the following statements is/are true about square matrix A of order n?

(A) $(-A)^{-1}$ is equal to $-A^{-1}$ which n is odd only

(B) If $A^n = O$, then $I + A + A^2 + ... + A^{n-1} = (I - A)^{-1}$

(C) If A is skew-symmetric matrix of odd order, then its inverse does not exist.

(D) $(A^T)^{-1} = (A^{-1})^T$ holds always

***96.** If A, B and C are three square matrices of the same order, then $AB = AC \Rightarrow B = C$. Then :

(A) $|A| \neq 0$ **(B)** A is invertible

(C) A may be orthogonal **(D)** A is symmetric

***97.** Suppose $a_1, a_2, \ldots$ are real numbers, with $a_1 \neq 0$. If $a_1, a_2, a_3, \ldots\ldots$ are in A.P., then :

(A) $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_5 & a_6 & a_7 \end{bmatrix}$ is singular (where $i = \sqrt{-1}$)

(B) the system of equations $a_1x + a_2y + a_3z = 0$, $a_4x + a_5y + a_6z = 0$, $a_7x + a_8y + a_9z = 0$ has infinite number of solutions

(C) $B = \begin{bmatrix} a_1 & ia_2 \\ ia_2 & a_1 \end{bmatrix}$ is nonsingular

(D) None of these

***98.** Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. Then which of following is not true ?

(A) $\lim\limits_{n \to \infty} \frac{1}{n^2} A^{-n} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$ **(B)** $\lim\limits_{n \to \infty} \frac{1}{n} A^{-n} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$

(C) $A^{-n} = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix} \forall n \neq N$ **(D)** None of these

***99.** If C is skew-symmetric matrix of order n and X is $n \times 1$ column matrix, then X^TCX is :

(A) singular **(B)** non-singular **(C)** invertible **(D)** non-invertible

***100.** If $S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $A = \begin{bmatrix} b+c & c+a & b-c \\ c-b & c+b & a-b \\ b-c & a-c & a+b \end{bmatrix} (a, b, c \neq 0)$, then SAS^{-1} is :

(A) symmetric matrix **(B)** diagonal matrix

(C) invertible matrix **(D)** singular matrix

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***101.** If $AB = A$ and $BA = B$, then :

(A) $A^2B = A^2$ **(B)** $B^2A = B^2$ **(C)** $ABA = A$ **(D)** $BAB = B$

102. Let K be a positive real number and

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} \text{ and } \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2 \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$$

If $\det(adj\, A) + \det(adj\, B) = 10^6$, then $[k]$ is equal to _____.

***103.** Let $A = a_{ij}$ be a matrix of order 3, where $a_{ij} = \begin{cases} x; & \text{if } i = j, x \in R \\ 1; & \text{if } |i-j| = 1; \\ 0; & \text{otherwise} \end{cases}$ then which of the following hold(s) good :

(A) for $x = 2$, A is a diagonal matrix

(B) A is a symmetric matrix

(C) for $x = 2$, $\det A$ has the value equal to 6

(D) Let $f(x) = \det A$, then the function $f(x)$ has both the maxima and minima

104. Let *M* and *N* be two 3×3 nonsingular skew-symmetric matrices such that *MN* = *NM*. If P^T denotes the transpose of *P*, then $M^2N^2\left(M^TN\right)^{-1}\left(MN^{-1}\right)^T$ is equal to :

(A) M^2 **(B)** $-N^2$ **(C)** $-M^2$ **(D)** MN

***105.** For 3×3 matrices *M* and *N*, which of the following statements (*s*) is (are) NOT correct ?

(A) N^TMN is symmetric or skew-symmetric, according as *M* is symmetric or skew- symmetric

(B) $MN - NM$ is skew-symmetric for all symmetric matrices *M* and *N*

(C) *MN* is symmetric for all symmetric matrices *M* And *N*

(D) $(adj\, M)(adj N) = adj(MN)$ for all invertible matrices *M* and *N*

106. Let $A = \left[a_{ij}\right]_{3\times3}$ be a matrix such that $AA^T = 4I$ and $a_{ij} + 2c_{ij} = 0$, where c_{ij} is the cofactor of a_{ij} and I is the unit matrix of order 3. $\begin{vmatrix} a_{11}+4 & a_{12} & a_{13} \\ a_{21} & a_{22}+4 & a_{23} \\ a_{31} & a_{32} & a_{33}+4 \end{vmatrix} + 5\lambda \begin{vmatrix} a_{11}+1 & a_{12} & a_{13} \\ a_{21} & a_{22}+1 & a_{23} \\ a_{31} & a_{32} & a_{33}+1 \end{vmatrix} = 0$ then the value of 10λ is ____.

***107.** Let A and B be two nonsingular square matrices, A^T and B^T are the transpose matrices of A and B, respectively, then which of the following are correct ?

(A) B^TAB is symmetric matrix if A is symmetric

(B) B^TAB is symmetric matrix if B is symmetric

(C) B^TAB is skew-symmetric matrix for every matrix A

(D) B^TAB is skew-symmetric matrix If A is skew-symmetric

108. If A is a symmetric and B skew-symmetric matrix and $A+B$ is nonsingular and $C=(A+B)^{-1}(A-B)$, then prove that :

(i) $C^T(A+B)C = A+B$ **(ii)** $C^T(A-B)C = A-B$ **(iii)** $C^TAC = A$

109. If $A = \begin{bmatrix} \operatorname{cosec}^2\alpha & 0 & 0 \\ 0 & \operatorname{cosec}^2\beta & 0 \\ 0 & 0 & \operatorname{cosec}^2\gamma \end{bmatrix}$ and $B = \begin{bmatrix} \sec^2\alpha & 0 & 0 \\ 0 & \sec^2\beta & 0 \\ 0 & 0 & \sec^2\gamma \end{bmatrix}$ where $\alpha,\beta,\gamma \in R - \left\{n\frac{\pi}{2} : n \in I\right\}$ and

$C = \left(A^{-5}+B^{-5}\right)+5A^{-1}B^{-1}\left(A^{-3}+B^{-3}\right)+10\left(A^{-1}+B^{-1}\right)A^{-2}B^{-2}$ (where $X^{-n} = (X^{-1})^n$, then $|C|$ =

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4

***110.** Matrix $A = \begin{bmatrix} a & -b & 0 \\ c & 0 & b \\ 0 & -c & -a \end{bmatrix}$ then :

(A) $A^9 = \left(a^2-2bc\right)^3 A$ **(B)** $A^9 = \left(a^2-2bc\right)^4 A$

(C) $\left|A^2+\left(2bc-a^2\right)I_3\right| = 0$ **(D)** $\left|A^2+\left(a^2-2bc\right)I_3\right| = 0$

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***111.** A is even ordered non singular symmetric matrix and B is even ordered non singular skew symmetric matrix such that $AB = BA$, then $A^3B^3(B'A)^{-1}\left(A^{-1}B^{-1}\right)'$ AB is equal to :

(A) A^2B^2 **(B)** B^2A^2 **(C)** $-A^2B^2$ **(D)** $-B^2A^2$

***112.** Let P be a 3×3 matrix such that $P^T = \lambda P + \mu I, \lambda, \mu \in R,$ where $\lambda \neq \pm 1, \ \mu \neq 0$ and P^T denotes transpose of matrix P then :

(A) P is singular matrix

(B) P is non singular matrix

(C) $P\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$ have a unique solution

(D) trace of $P = \dfrac{\mu}{1-\lambda}$

***113.** Let M and N be two 3 × 3 matrices such that $MN = NM$, $M^3 = N^6$ and $M \neq N^2$ then :

(A) $\left|M^2N^2 + M^3 + MN^4\right| = 0$

(B) There exist a non zero 3 ×1 matrix U such that $\left(M^2N^2 + M^3 + MN^4\right)U$ is a zero matrix

(C) $\left|M^2N^2 + M^3 + MN^4\right| \geq 1$

(D) For a 3 × 1 matrix U such that $\left(M^2N^2 + M^3 + MN^4\right)U$ is a zero matrix then U is a zero matrix

***114.** Consider the system of equations $3x - y + 4z = 3$, $x + 2y - 3z = -2$, $6x + 5y + \lambda z = -3$, then for $\lambda = -5$:

(A) system has no solution

(B) system has infinitely many solutions lying on a line $\dfrac{7x-4}{-5} = \dfrac{7y+9}{13} = z$

(C) system has infinitely many solutions lying on a line $\dfrac{7x+1}{-5} = \dfrac{7y-4}{13} = \dfrac{z-1}{1}$

(D) system has infinitely many solutions representing a plane

115. Suppose $a, b, c \in R$ and $abc = 1$. If $A = \begin{pmatrix} 2a & b & c \\ b & 2c & a \\ c & a & 2b \end{pmatrix}$ such that $A'A = 4^{\frac{1}{3}}I$ and $|A| > 4$, find $[a^3 + b^3 + c^3]$. [.] denotes greatest integer function.

116. $A = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$ and $A^8 + A^6 + A^4 + A^2 + (I)V = \begin{bmatrix} 0 \\ 11 \end{bmatrix}$ (where I is the 2×2 identity matrix), then the product of all elements of matrix V is _______.

117. If $\begin{bmatrix} a & b \\ c & 1-a \end{bmatrix}$ is an idempotent matrix and $f(x) = x - x^2$; $bc = 1/4$, then the value of $1/f(a)$ is ______.

118. Let X be the solution set of the equation $A^X = I$, where $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ and I is the corresponding unit matrix and $x \subseteq N$, then the minimum value of $\sum(\cos^X \theta + \sin^X \theta)$, $\theta \in R$.

119. $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$ and $f(x)$ is defined as $f(x) = \det.(A^T A^{-1})$ then the value of $\underbrace{f(f(f(f.......f(x))))}_{n \text{ times}}$ is $(n \geq 2)$ _______.

120. If A is an idempotent matrix satisfying, $(I - 0.4A)^{-1} = I - \alpha A$, where I is the unit matrix of the same order as that of A, then the value of $|9\alpha|$ is equal to ______.

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121. Let $A=\begin{bmatrix}3x^2\\1\\6x\end{bmatrix}, B=[a\ b\ c]$, and $C=\begin{bmatrix}(x+2)^2 & 5x^2 & 2x\\5x^2 & 2x & (x+2)^2\\2x & (x+2)^2 & 5x^2\end{bmatrix}$ be three given matrices, where a,b,c and $x\in R$. Given that $tr(AB)=tr(C)x\in R$, where $tr(A)$ denotes trace of A. If $f(x)=ax^2+bx+c$, then the value of $f(1)$ is ______.

122. Let $A=[a_{ij}]_{3\times 3}$ be a matrix such that $AA^T=4I$ and $a_{ij}+2c_{ij}=0$, where c_{ij} is the cofactor of a_{ij} and I is the unit matrix of order 3.

$$\begin{vmatrix}a_{11}+4 & a_{12} & a_{13}\\a_{21} & a_{22}+4 & a_{23}\\a_{31} & a_{32} & a_{33}+4\end{vmatrix}+5\lambda\begin{vmatrix}a_{11}+1 & a_{12} & a_{13}\\a_{21} & a_{22}+1 & a_{23}\\a_{31} & a_{32} & a_{33}+1\end{vmatrix}=0$$

Then the value of 10λ is _______.

123. Let S be the set which contains all possible values of l,m,n,p,q,r for which

$A=\begin{bmatrix}l^2-3 & p & 0\\0 & m^2-8 & q\\r & 0 & n^2-15\end{bmatrix}$ be a nonsingular idempotent matrix. Then the sum of all the elements of the set S is ______.

124. Let α,β,γ are the real roots of the equation $x^3+ax^2+bx+c=0\,(a,b,c\in R$ and $a\neq 0)$. If the system of equations (in u, v and w) given by

$$\alpha u+\beta v+\gamma w=0$$
$$\beta u+\gamma v+\alpha w=0$$
$$\gamma u+\alpha v+\beta w=0$$

has non-trivial solutions, then the value of a^2/b is _______.

125. If $a_1,a_2,a_3,5,4,a_6,a_7,a_8,a_9$ are in H.P., and $D=\begin{vmatrix}a_1 & a_2 & a_3\\5 & 4 & a_6\\a_7 & a_8 & a_9\end{vmatrix}$, then the value of [D] is (where [.] represents the greatest integer function)

126. If $\begin{vmatrix}(\beta+\gamma-\alpha-\delta)^4 & (\beta+\gamma-\alpha-\delta)^2 & 1\\(\gamma+\alpha-\beta-\delta)^4 & (\gamma+\alpha-\beta-\delta)^2 & 1\\(\alpha+\beta-\gamma-\delta)^4 & (\alpha+\beta-\gamma-\delta)^2 & 1\end{vmatrix}=-k(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\beta-\gamma)(\beta-\delta)(\gamma-\delta)$, then the value of $(k)^{1/2}$ is _______.

127. Absolute value of sum of roots of the equation $\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 2x+3 & 3x+4 & 4x+5 \\ 3x+5 & 5x+8 & 10x+17 \end{vmatrix} = 0$ is _______.

128. Let $A_k(x)$ denote a polynomial of degree k where $k \geq 1$, & $\Delta(x) = \begin{vmatrix} A_3'(x) & (xA_2(x))' & (x^2A_1(x))' \\ A_3''(x) & (xA_2(x))'' & (x^2A_1(x))'' \\ A_3'''(x) & (xA_2(x))''' & (x^2A_1(x))''' \end{vmatrix}$

find $\Delta'(100!)$

129. If α, β, γ are real numbers, then without expanding at any stage, show that

$$\begin{vmatrix} 1 & \cos(\beta-\alpha) & \cos(\gamma-\alpha) \\ \cos(\alpha-\beta) & 1 & \cos(\gamma-\beta) \\ \cos(\alpha-\gamma) & \cos(\beta-\gamma) & 1 \end{vmatrix} = 0 .$$

130. Prove that $\begin{vmatrix} (a-x)^2 & (a-y)^2 & (a-z)^2 \\ (b-x)^2 & (b-y)^2 & (b-z)^2 \\ (c-x)^2 & (c-y)^2 & (c-z)^2 \end{vmatrix} = \begin{vmatrix} (1+ax)^2 & (1+bx)^2 & (1+cx)^2 \\ (1+ay)^2 & (1+by)^2 & (1+cy)^2 \\ (1+az)^2 & (1+bz)^2 & (1+cz)^2 \end{vmatrix}$

$= 2(b-c)(c-a)(a-b) \times (y-z)(z-x)(x-y)$.

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131. Express $\Delta = \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix}$ as square of a determinant and hence evaluate it:

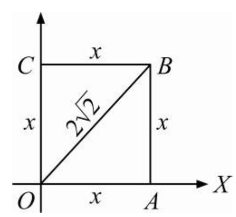
132. Prove without expansion that $\begin{vmatrix} ah+bg & g & ab+ch \\ bf+ba & f & hb+bc \\ af+bc & c & bg+fc \end{vmatrix} = a\begin{vmatrix} ah+bg & a & h \\ bf+ba & h & b \\ af+bc & g & f \end{vmatrix}$

133. If $y = \begin{vmatrix} \sin x & \cos x & \sin x \\ \cos x & -\sin x & \cos x \\ x & 1 & 1 \end{vmatrix}$, find $\dfrac{dy}{dx}$.

134. If $f(x) = \begin{vmatrix} x^n & n! & 2 \\ \cos x & \cos\dfrac{n\pi}{2} & 4 \\ \sin x & \sin\dfrac{n\pi}{2} & 8 \end{vmatrix}$, then find the value of $\dfrac{d^n}{dx^n}[f(x)]_{x=0} \cdot (n \in Z)$.

135. Show that $\begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix} = \begin{vmatrix} a^2 & c^2 & 2ac-b^2 \\ 2ab-c^2 & b^2 & a^2 \\ b^2 & 2bc-a^2 & c^2 \end{vmatrix}$

136. Write down the 2×2 matrix A which corresponds to a counterclockwise rotation of 60° about the origin. In figure, the square $OABC$ has it diagonal OB of $2\sqrt{2}$ units in length. The square is rotated counterclockwise about O through 60°. Find the coordinates of the vertices of the square after rotating.



137. Let x, y, z be the distinct common roots of equations $a^{10} = 1$ and $a^{15} = 1$ such that there real part is positive and $\omega = \begin{vmatrix} 1+x^2+x^4 & 1+xy+x^2y^2 & 1+xz+x^2z^2 \\ 1+xy+x^2y^2 & 1+y^2+y^4 & 1+yz+y^2z^2 \\ 1+xz+x^2z^2 & 1+yz+y^2z^2 & 1+z^2+z^4 \end{vmatrix}$ then :

(A) ω is purely real
(B) ω is purely imaginary
(C) $\text{Re}(\omega) > 0$
(D) $\text{Re}(\omega) < 0$

138. If A, B and C are the angles of a triangle, show that the system of equations $x\sin 2A + y\sin C + z\sin B = 0$, $x\sin C + y\sin 2B + z\sin A = 0$, and $x\sin B + y\sin A + z\sin 2C = 0$ possesses nontrivial solution. Hence, system has infinite solutions.

139. If $ax_1^2 + by_1^2 + cz_1^2 = ax_2^2 + by_2^2 + cz_2^2 = ax_3^2 + by_3^2 + cz_3^2 = d$, $ax_2x_3 + by_2y_3 + cz_2z_3 = ax_3x_1 + by_3y_1 + cz_3z_1 = ax_1x_2 + by_1y_2 + cz_1z_2 = f$, then prove that $\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = (d - f)\left\{\frac{(d + 2f)}{abc}\right\}^{1/2}$

140. Let $\Delta = \begin{vmatrix} 2a_1b_1 & a_1b_2 + a_2b_1 & a_1b_3 + a_3b_1 \\ a_1b_2 + a_2b_1 & 2a_2b_2 & a_2b_3 + a_3b_2 \\ a_1b_3 + a_3b_1 & a_3b_2 + a_2b_3 & 2a_3b_3 \end{vmatrix}$. Expressing Δ as the product of two determinants, show that $\Delta = 0$ hence, show that if $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = (lx + my + n)(l'x + m'y + n')$, then $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$.

Matrices and Determinants

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141. If $a^2+b^2+c^2=-2$ and $f(x)=\begin{vmatrix}1+a^2x & (1+b^2)x & (1+c^2)x\\(1+a^2)x & 1+b^2x & (1+c^2)x\\(1+a^2)x & (1+b^2)x & 1+c^2x\end{vmatrix}$, then find the degree of polynomial $f(x)$.

polynomial of degree.......

142. If the system of equations $3x-2y+z=0$, $\lambda x-14y+15z=0$, $x+2y+3z=0$ has a non-trivial solution, then find the value of λ^2.

143. Let $f(x)=\begin{vmatrix}\cos x & -x & 1\\2\sin x & -x^2 & 2x\\\tan x & -x & 1\end{vmatrix}$. Find the value of $\lim\limits_{x\to 0}\frac{f'(x)}{x}$.

144. Let $f(x)=\begin{vmatrix}2\cos^2 x & \sin(2x) & -\sin x\\\sin(2x) & 2\sin^2 x & \cos x\\\sin x & -\cos x & 0\end{vmatrix}$. Find the value of $\frac{1}{\pi}\int_0^{\pi/2}[f(x)+f'(x)]dx$.

145. If $\begin{vmatrix}3a & -a+b & -a+c\\-b+a & 3b & -b+c\\-c+a & -c+b & 3c\end{vmatrix}=k(a+b+c)(ab+bc+ac)$, then find the value of k.

146. Let $\phi_1(x)=x+a_1$, $\phi_2(x)=x^2+b_1x+b_2$, $x_1=2, x_2=3$ and $x_3=5$ and $\Delta=\begin{vmatrix}1 & 1 & 1\\\phi_1(x_1) & \phi_1(x_2) & \phi_1(x_3)\\\phi_2(x_1) & \phi_2(x_2) & \phi_2(x_3)\end{vmatrix}$.

Find the value of Δ.

147. If the system of equations

$ax+hy+g=0$ ...(i)

$hx+by+f=0$ ...(ii)

and $ax^2+2hxy+by^2+2gx+2fy+c+t=0$ ...(iii)

has a unique solution and $\frac{abc+2fgh-af^2-bg^2-ch^2}{h^2-ab}=8$, find the value of '$t$'.

148. If $\begin{vmatrix}1 & x & x^2\\x & x^2 & 1\\x^2 & 1 & x\end{vmatrix}=3$, then the value of $\begin{vmatrix}x^3-1 & 0 & x-x^4\\0 & x-x^4 & x^3-1\\x-x^4 & x^3-1 & 0\end{vmatrix}$ is ......

149. If $A^2 = 3A - 2I$ and $A^8 = pA + qI$, then $p + q$ is ......

150. If A, B and C are $n \times n$ matrices and det(A) = 2, det(B) = 3 and det(C) = 5, then find the value of $= [\det(A^2BC^{-1})]$ (where $[\cdot]$ represents greatest integer function).

151. If matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, where a, b, c are real positive numbers, abc = 1 and $A^T A = I$, then find greatest value of $a^3 + b^3 + c^3$.

152. Let $a_r = r(^7C_r), b_r = (7-r)(^7C_r)$ and $A_r = \begin{bmatrix} a_r & 0 \\ 0 & b_r \end{bmatrix}$. If $A = \sum_{r=0}^{7} A_r = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, then find the value of $a + b$.

153. If the system of equations

$x + y + z = 5$

$x + 2y + 3z = 9$

$x + 3y + \alpha z = \beta$

has infinitely many solutions, then find $\beta - \alpha$

154. Let A be the set of all 3 × 3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0. If n is the number of such matrices, then find $\frac{n}{2}$.

155. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}$. If $A^{-1} = xA + yI$, then the value of $2y + x$, is ......

Matrices and Determinants

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1. If the system of linear equations

$$x + ky + 3z = 0$$
$$3x + ky - 2z = 0$$
$$2x + 4y - 3z = 0$$

has a non-zero solutions (x, y, z), then $\frac{xz}{y^2}$ is equal to :

(A) 30 **(B)** –10 **(C)** 10 **(D)** –30

2. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$, then the ordered pair (A, B) is equal to :

(A) (4, 5) **(B)** (–4, –5) **(C)** (–4, 3) **(D)** (–4, 5)

3. Let A be a matrix such that $A \cdot \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is a scalar matrix and $|3A| = 108$. Then A^2 equals :

(A) $\begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$ **(B)** $\begin{bmatrix} 4 & 0 \\ -32 & 36 \end{bmatrix}$ **(C)** $\begin{bmatrix} 4 & -32 \\ 0 & 36 \end{bmatrix}$ **(D)** $\begin{bmatrix} 36 & 0 \\ -32 & 4 \end{bmatrix}$

4. Let S be the set of all real values of k for which the system of linear equations $x + y + z = 2$; $2x + y - z = 3$; $3x + 2y + kz = 4$ has a unique solution. Then S is :

(A) an empty set **(B)** equal to R **(C)** equal to $\{0\}$ **(D)** equal to $R - \{0\}$

5. If the system of linear equations : $x + ay + z = 3$, $x + 2y + 2z = 6$, $x + 5y + 3z = b$ has no solution, then :

(A) $a = -1, b = 9$ **(B)** $a \neq -1,\ b = 9$ **(C)** $a = 1, b \neq 9$ **(D)** $a = -1, b \neq 9$

6. Suppose A is any 3 × 3 non-singular matrix and $(A - 3I)(A - 5I) = O$, where $I = I_3$ and $O = O_3$. If $\alpha A + \beta A^{-1} = 4I$, then $\alpha + \beta$ is equal to :

(A) 13 **(B)** 7 **(C)** 12 **(D)** 8

7. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = A^{20}$. Then the sum of the elements of the first column of B is :

(A) 211 **(B)** 251 **(C)** 231 **(D)** 210

8. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then $adj(3A^2 + 12A)$ is equal to :

(A) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$ **(B)** $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$ **(C)** $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$ **(D)** $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

9. If S is the set of distinct values of 'b' for which the following system of linear equations $x+y+z=1$, $x+ay+z=1$, $ax+by+z=0$ has no solution then S is :

(A) an infinite set

(B) a finite set containing two or more elements

(C) a singleton

(D) an empty set

10. If $S=\left\{x\in[0,2\pi]:\begin{vmatrix}0 & \cos x & -\sin x\\ \sin x & 0 & \cos x\\ \cos x & \sin x & 0\end{vmatrix}=0\right\}$, then $\sum_{x\in S}\tan\left(\frac{\pi}{3}+x\right)$ is equal to :

(A) $-2+\sqrt{3}$ **(B)** $4+2\sqrt{3}$ **(C)** $-4-2\sqrt{3}$ **(D)** $-2-\sqrt{3}$

11. The number of real values of λ for which the system of linear equations $2x+4y-\lambda z=0$, $4x+\lambda y+2z=0$, $\lambda x+2y+2z=0$ has infinitely many solutions, is :

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

12. Let A be any 3 × 3 invertible matrix. Then which one of the following is not always true ?

(A) $adj\ (adj\ (A))=|A|\cdot(adj\,(A))^{-1}$ **(B)** $adj\ (adj\ (A))=|A|^2\cdot(adj\,(A))^{-1}$

(C) $adj\ (A)=|A|\cdot A^{-1}$ **(D)** $adj\ (adj\ (A))=|A|\cdot A$

13. For two 3 × 3 matrices A and B, let $A+B=2B'$ and $3A+2B=I_3$, where B' is the transpose of and I_3 is 3×3 identity matrix. Then :

(A) $10A+5B=3I_3$ **(B)** $5A+10B=2I_3$ **(C)** $3A+6B=2I_3$ **(D)** $B+2A=I_3$

14. If $A=\begin{bmatrix}5a & -b\\ 3 & 2\end{bmatrix}$ and $A\,adj\ A-AA^T$, then $5a+b$ is equal to :

(A) –1 **(B)** 5 **(C)** 4 **(D)** 13

15. The system of linear equation $x+\lambda y-z=0, \lambda x-y-z=0, x+y-\lambda z=0$ has a non-trivial solution for :

(A) infinitely many values of λ **(B)** exactly one value of λ

(C) exactly two values of λ **(D)** exactly three values of λ

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16. The number of distinct real roots of the equation, $\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$ in the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ is :

(A) 1 **(B)** 4 **(C)** 2 **(D)** 3

17. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P^T Q^{2015} P$ is :

(A) $\begin{bmatrix} 0 & 2015 \\ 0 & 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} 2015 & 0 \\ 1 & 2015 \end{bmatrix}$ **(C)** $\begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$ **(D)** $\begin{bmatrix} 2015 & 1 \\ 0 & 2015 \end{bmatrix}$

18. Let A be a 3 × 3 matrix such that $A^2 - 5A + 7I = O$.

Statement -I : $A^{-} = \frac{1}{7}(5I - A)$.

Statement -II: The polynomial $A^3 - 2A^2 - 3A + I$ can be reduced to $5(A - 4I)$. Then :

(A) Both the statements are true.

(B) Both the statements are false.

(C) Statement-I is true, but Statement -II is false.

(D) Statement-I is false, but Statement-II is true.

19. If $A = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}$, then the determinant of the matrix $(A^{2016} - 2A^{2015} - A^{2014})$ is :

(A) –175 **(B)** 2014 **(C)** 2016 **(D)** –25

20. The set of all values of λ for which the system of linear equations

$2x_1 - 2x_2 + x_3 = \lambda x_1,\ 2x_1 - 3x_2 + 2x_3 = \lambda x_2,\ -x_1 + 2x_2 = \lambda x_3$ has a non-trivial solution,

(A) contains two elements **(B)** contains more than two elements

(C) is an empty set **(D)** is a singleton

21. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is a 3 × 3 identify matrix, then the ordered pair (a, b) is equal to :

(A) (2, 1) **(B)** (–2, –1) **(C)** (2, –1) **(D)** (–2, 1)

22. The least value of the product xyz for which the determinant $\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix}$ is non-negative is :

(A) $-2\sqrt{2}$ **(B)** $-16\sqrt{2}$ **(C)** -8 **(D)** -1

23. If $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then which one of the following statements is not correct ?

(A) $A^4 - I = A^2 + I$ **(B)** $A^3 - I = A(A - I)$

(C) $A^2 + I = A(A^2 - I)$ **(D)** $A^3 + I = A(A^3 - I)$

24. If A is a 3 × 3 matrix such that $|5 \cdot adj\, A| = 5$, then $|A|$ is equal to :

(A) $\pm\frac{1}{5}$ **(B)** ± 5 **(C)** ± 1 **(D)** $\pm\frac{1}{25}$

25. If $\begin{vmatrix} x^2 + x & x+1 & x-2 \\ 2x^2 + 3x - 1 & 3x & 3x - 3 \\ x^2 + 2x + 3 & 2x - 1 & 2x - 1 \end{vmatrix} = ax - 12$, then '$a$' is equal to :

(A) 12 **(B)** 24 **(C)** –12 **(D)** –24

26. If A is an 3 × 3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' equals :

(A) I **(B)** B^{-1} **(C)** $(B^{-1})'$ **(D)** $I + B$

27. If $\alpha, \beta \neq 0$ and $f(n) = \alpha^n + \beta^n$ and $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$, then K is equal to :

(A) $\frac{1}{\alpha\beta}$ **(B)** 1 **(C)** –1 **(D)** $\alpha\beta$

28. If A an 3×3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' is equal to :

(A) $I + B$ **(B)** I **(C)** B^{-1} **(D)** $\left(B^{-1}\right)'$

29. The number of values of k, for which the system of equations $(k+1)x + 8y = 4k$ **,** $kx + (k+3)y = 3k - 1$ has no solution, is :

(A) ∞ **(B)** 1 **(C)** 2 **(D)** 3

30. If $P\begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to :

(A) 4 **(B)** 11 **(C)** 5 **(D)** 0

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31. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$. If μ_1 and μ_2 are column matrices such that $Au_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $Au_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, then $\mu_1 + \mu_2$ is equal to :

(A) $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$ **(C)** $\begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$ **(D)** $\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

32. Let P and Q be 3×3 matrices $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to:

(A) –2 **(B)** 1 **(C)** 0 **(D)** -1

33. The number of values of k for which the linear equations
$4x + ky + 2z = 0, kx + 4y + z = 0$ and $2x + 2y + z = 0$ posses a non-zero solution, is :

(A) 2 **(B)** 1 **(C)** 0 **(D)** 3

34. Let A and B be two symmetric matrices of order 3.

Statement I : A(BA) and (AB) A are symmetric matrices.

Statement II : AB is symmetric matrix, if matrix multiplication of A with B is commutative.

(A) Statement I is true, Statement II is true; Statement II is not a correct explanation of Statement I
(B) Statement I is true, Statement II is false
(C) Statement I is false, Statement II is true
(D) Statement I is true, Statement II is true; Statement II is a correct explanation of Statement I

35. If the trivial solution is the only solution of the system of equations $x - ky + z = 0, kx + 3y - kz = 0$ and $3x + y - z = 0$. Then, set of all values of k is :

(A) $\{2, -3\}$ **(B)** $R - \{2, -3\}$ **(C)** $R - \{2\}$ **(D)** $R - \{-3\}$

36. **Statement I :** Determinant of a skew-symmetric matrix of order 3 is zero.

Statement II : For any matrix A, $\det(A^T) = \det(A)$ *and* $\det(-A) = -\det(A)$. Then :

(A) Statement I is true and Statement II is false
(B) Both Statements are true
(C) Both Statements are false
(D) Statement I is false and Statement II is true

37. If $\omega \neq 1$ is the complex cube root of unity and matrix $H = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then H^{70} is equal to :

(A) H **(B)** 0 **(C)** $-H$ **(D)** H^2

38. Consider the system of linear equations $x_1 + 2x_2 + x_3 = 3, 2x_1 + 3x_2 + x_3 = 3$ and $3x_1 + 5x_2 + 2x_3 = 1$. Then system has :

(A) Infinite number of solutions **(B)** Exactly 3 solutions
(C) A unique solution **(D)** No solution

39. The number of 3×3 non-singular matrices, with four entries as 1 and all other entries as 0, is :

(A) Less than 4 **(B)** 5 **(C)** 6 **(D)** Atleast 7

40. Let A be 2×2 matrix with non-zero entries and $A^2 = I$, where I is 2×2 identity matrix.
Define $tr(A) =$ Sum of diagonal elements of A and $|A| =$ Determinant of matrix A.

Statement I : $tr(A) = 0$

Statement II : $|A| = 1$

(A) Statement I is false, Statement II is true
(B) Statement I is true, Statement II is true; Statement II is a correct explanation of Statement I
(C) Statement I is true, Statement II is true; Statement II is not a correct explanation of Statement I
(D) Statement I is true, Statement II is false

41. Let a, b and c be such that $(b+c) \neq 0$. If $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^{n}c \end{vmatrix} = 0$, then the value of '$n$' is :

(A) Zero **(B)** Any even integer
(C) Any odd integer **(D)** Any integer

42. Let A be 2×2 matrix.

Statement I : $adj(adj\,A) = A$

Statement II : $|adj\,A| = A$

(A) Statement I is false, Statement II is true
(B) Statement I is true, Statement II is true; Statement II is a correct explanation of Statement I
(C) Statement I is true, Statement II is true; Statement II is not a correct explanation of Statement I
(D) Statement I is true, Statement II is false

43. Let A is 2×2 matrix with real entries. Let I be the 2×2 identity matrix. Denote by $tr(A)$, the sum of diagonal entries of A. Assume that $A^2 = I$.

Statement I : If $A \neq I$ and $A \neq -I$, then $\det(A) = -1$

Statement II : If $A \neq I$ and $A \neq -I$, then $tr(A) \neq 0$

(A) Statement I is false, Statement II is true
(B) Statement I is true, Statement II is true; Statement II is a correct explanation of Statement I
(C) Statement I is true, Statement II is true; Statement II is not a correct explanation of Statement I
(D) Statement I is true, Statement II is false

44. Let a, b and c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz$, $y = az + cx$ and $z = bx + ay$. Then, $a^2 + b^2 + c^2 + 2abc$ is equal to :

(A) 1 **(B)** 2 **(C)** -1 **(D)** 0

45. Let A be a square matrix all of whose entries are integers. Then, which one of the following is true ?

(A) If $\det(A) = \pm 1$, then A^{-1} need not exist
(B) If $\det(A) = \pm 1$, then A^{-1} exist but all its entries are not necessarily integers
(C) If $\det(A) = \pm 1$, then A^{-1} exist but all its entries are non-integers
(D) If $\det(A) = \pm 1$, then A^{-1} exist but all its entries are integers

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46. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1-y \end{vmatrix}$ for $x \neq 0, y \neq 0$, then D is :

(A) Divisible by neither x nor y **(B)** Divisible by both x and y
(C) Divisible by x but not y **(D)** Divisible by y but not x

47. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$. If $|A^2| = 25$, then $|\alpha|$ is equal to :

(A) 5^2 **(B)** 1 **(C)** $\frac{1}{5}$ **(D)** 5

48. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A-B)(A+B)$, then which of the following will be always true ?

(A) $AB = BA$ **(B)** Either of A or B is a zero matrix
(C) Either of A or B is an identity matrix **(D)** $A = B$

49. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$; $a, b \in N$. Then :

(A) There exists more than one but finite number of B's such that $AB = BA$
(B) There exists exactly one B such that $AB = BA$
(C) There exist infinitely many B's such that $AB = BA$
(D) There cannot exist any B such that $AB = BA$

50. If $A^2 - A + I = O$, then the inverse of A is :

(A) $I - A$ **(B)** $A - I$ **(C)** A **(D)** $A + I$

51. If $a^2 + b^2 + c^2 = -2$ and $f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$ then $f(x)$ is a polynomial of degree :

(A) 2 **(B)** 3 **(C)** 0 **(D)** 1

52. The system of equations $\alpha x + y + z = \alpha - 1, x + \alpha y + z = \alpha - 1$ and $x + y + \alpha z = \alpha - 1$ has no solution, if α is :

(A) 1 **(B)** not -2 **(C)** Either –2 or 1 **(D)** –2

53. If $a_1, a_2, ..., a_n, ...$ are in GP, then the determinant $\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$ is equal to :

(A) 2 **(B)** 4 **(C)** 0 **(D)** 1

54. Let $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$. The only correct statement about the matrix A is :

(A) A is a zero matrix **(B)** $A = (-1)I$, where I is a unit matrix

(C) A^{-1} does not exist **(D)** $A^2 = I$

55. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$. If B is the inverse of matrix A, then α is equal to :

(A) -2 **(B)** 1 **(C)** 2 **(D)** 5

56. If the system of linear equations $x + 2ay + az = 0$, $x + 3by + bz = 0$ and $x + 4cy + cz = 0$ has a non-zero solution, then a, b and c :

(A) are in AP **(B)** are in GP **(C)** are in HP **(D)** satisfy $a + 2b + 3c = 0$

57. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$:

(A) $\alpha = a^2 + b^2$ and $\beta = ab$ **(B)** $\alpha = a^2 + b^2$ and $\beta = 2ab$

(C) $\alpha = a^2 + b^2$ and $\beta = a^2 - b^2$ **(D)** $\alpha = 2ab$ and $\beta = a^2 + b^2$

58. If 1, ω and ω^2 are the cube roots of unity, then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to :

(A) 0 **(B)** 1 **(C)** ω **(D)** ω^2

59. If l, m and n are the pth, qth and rth terms of a GP and all positive, then $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ is :

(A) 3 **(B)** 2 **(C)** 1 **(D)** 0

60. If $\omega(\neq 1)$ is a cubic root of unity, then $\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2 - 1 \\ -i & -1+\omega-i & -1 \end{vmatrix}$ is equal to :

(A) 0 **(B)** 1 **(C)** i **(D)** ω

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61. If A is a symmetric matrix and B is a skew-symmetric matrix such that $A+B=\begin{bmatrix}2 & 3\\ 5 & -1\end{bmatrix}$, then AB is equal to:

(A) $\begin{bmatrix}-4 & -2\\ -1 & 4\end{bmatrix}$ **(B)** $\begin{bmatrix}4 & -2\\ -1 & -4\end{bmatrix}$ **(C)** $\begin{bmatrix}4 & -2\\ 1 & -4\end{bmatrix}$ **(D)** $\begin{bmatrix}-4 & 2\\ 1 & 4\end{bmatrix}$

62. The total number of matrices $A=\begin{bmatrix}0 & 2y & 1\\ 2x & y & -1\\ 2x & -y & 1\end{bmatrix},(x,y\in R, x\neq y)$ for which $A^TA=3I_3$ is:

(A) 2 **(B)** 4 **(C)** 3 **(D)** 6

63. Let $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix},(\alpha\in R)$ such that $A^{32}=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$. Then, a value of α is:

(A) $\frac{\pi}{32}$ **(B)** 0 **(C)** $\frac{\pi}{64}$ **(D)** $\frac{\pi}{16}$

64. Let $P=\begin{bmatrix}1 & 0 & 0\\ 3 & 1 & 0\\ 9 & 3 & 1\end{bmatrix}$ and $Q=[q_{ij}]$ be two 3 × 3 matrices such that $Q-P^5=I_3$. Then, $\frac{q_{21}+q_{31}}{q_{32}}$ is equal to:

(A) 10 **(B)** 135 **(C)** 9 **(D)** 15

65. Let $A=\begin{bmatrix}0 & 2q & r\\ p & q & -r\\ p & -q & r\end{bmatrix}$. If $AA^T=I_3$, then $|p|$ is:

(A) $\frac{1}{\sqrt{5}}$ **(B)** $\frac{1}{\sqrt{2}}$ **(C)** $\frac{1}{\sqrt{3}}$ **(D)** $\frac{1}{\sqrt{6}}$

66. A value of $\theta\in(0,\pi/3)$, for which $\begin{vmatrix}1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta\\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta\\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta\end{vmatrix}=0$, is:

(A) $\frac{\pi}{9}$ **(B)** $\frac{\pi}{18}$ **(C)** $\frac{7\pi}{24}$ **(D)** $\frac{7\pi}{36}$

67. The sum of the real roots of the equation $\begin{vmatrix}x & -6 & -1\\ 2 & -3x & x-3\\ -3 & 2x & x+2\end{vmatrix}=0$, is equal to:

(A) 0 **(B)** –4 **(C)** 6 **(D)** 1

68. If $\Delta_1=\begin{vmatrix}x & \sin\theta & \cos\theta\\ -\sin\theta & -x & 1\\ \cos\theta & 1 & x\end{vmatrix}$ and $\Delta_2=\begin{vmatrix}x & \sin 2\theta & \cos 2\theta\\ -\sin 2\theta & -x & 1\\ \cos 2\theta & 1 & x\end{vmatrix}, x\neq 0$, then for all $\theta\in\left(0,\frac{\pi}{2}\right)$

(A) $\Delta_1+\Delta_2=-2(x^3+x-1)$ **(B)** $\Delta_1-\Delta_2=-2x^3$

(C) $\Delta_1+\Delta_2=-2x^3$ **(D)** $\Delta_1-\Delta_2=x(\cos 2\theta-\cos 4\theta)$

69. If $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\cdots\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$, then the inverse of $\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$ is:

(A) $\begin{bmatrix}1 & 0\\12 & 1\end{bmatrix}$ **(B)** $\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$ **(C)** $\begin{bmatrix}1 & 0\\13 & 1\end{bmatrix}$ **(D)** $\begin{bmatrix}1 & -12\\0 & 1\end{bmatrix}$

70. Let α and β be the roots of the equation $x^2+x+1=0$. Then, for $y\neq 0$ in R, $\begin{vmatrix}y+1 & \alpha & \beta\\ \alpha & y+\beta & 1\\ \beta & 1 & y+\alpha\end{vmatrix}$ is equal to:

(A) $y(y^2-1)$ **(B)** $y(y^2-3)$ **(C)** y^3-1 **(D)** y^3

71. Let the numbers 2, b, c be in an AP and $A=\begin{bmatrix}1 & 1 & 1\\2 & b & c\\4 & b^2 & c^2\end{bmatrix}$. If $\det(A)\in[2,16]$, then c lies in the interval.

(A) $[3, 2+2^{3/4}]$ **(B)** $(2+2^{3/4}, 4)$ **(C)** $[4, 6]$ **(D)** $[2, 3)$

72. If $A=\begin{bmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{bmatrix}$; then for all $\theta\in\left(\frac{3\pi}{4},\frac{5\pi}{4}\right)$, $\det(A)$ lies in the interval.

(A) $\left(\frac{3}{2},3\right]$ **(B)** $\left[\frac{5}{2},4\right)$ **(C)** $\left(0,\frac{3}{2}\right]$ **(D)** $\left(1,\frac{5}{2}\right]$

73. If $\begin{vmatrix}a-b-c & 2a & 2a\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}=(a+b+c)(x+a+b+c)^2, x\neq 0$ and $a+b+c\neq 0$, then x is equal to:

(A) $-(a+b+c)$ **(B)** $-2(a+b+c)$ **(C)** $2(a+b+c)$ **(D)** abc

74. Let $a_1, a_2, a_3 \ldots\ldots, a_{10}$ be in GP with $a_i>0$ for i = 1, 2, ...,10 and S be the set of pairs (r,k), where $k\in N$ (the set of natural numbers) for which

$$\begin{vmatrix}\log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k\\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k\\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k\end{vmatrix}=0$$

Then, the number of elements in S, is:

(A) 4 **(B)** 2 **(C)** 10 **(D)** Infinitely many

75. Let $A=\begin{bmatrix}2 & b & 1\\ b & b^2+1 & b\\ 1 & b & 2\end{bmatrix}$, where $b>0$. Then, the minimum value of $\frac{\det(A)}{b}$ is:

(A) $-\sqrt{3}$ **(B)** $-2\sqrt{3}$ **(C)** $2\sqrt{3}$ **(D)** $\sqrt{3}$

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76. Let $d \in R$, and $A = \begin{bmatrix} -2 & 4+d & (\sin\theta)-2 \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d \end{bmatrix}, \theta \in [0, 2\pi]$. If the minimum value of det(A) is 8, then a value of d is:

(A) -5 **(B)** -7 **(C)** $2(\sqrt{2}+1)$ **(D)** $2(\sqrt{2}+2)$

77. If $B = \begin{bmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{bmatrix}$ is the inverse of a 3×3 matrix A, then the sum of all values of α for which det(A)+1= 0, is:

(A) 0 **(B)** -1 **(C)** 1 **(D)** 2

78. If $A = \begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$ then A is:

(A) Invertible only when $t = \pi$ **(B)** Invertible for every $t \in R$

(C) Not invertible for any $t \in R$ **(D)** Invertible only when $t = \frac{\pi}{2}$

79. Let A and B be two invertible matrices of order 3 × 3. If $\det(ABA^T) = 8$ and $\det(AB^{-1}) = B$, then $\det(BA^{-1}B^T)$ is equal to:

(A) 1 **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{16}$ **(D)** 16

80. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$, is equal to:

(A) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ **(B)** $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ **(C)** $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ **(D)** $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

81. If $[x]$ denotes the greatest integer $\le x$, then the system of linear equations $[\sin\theta]x + [-\cos\theta]y = 0$, $[\cot\theta]x + y = 0$.

(A) Have infinitely many solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and has a unique solution of $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

(B) Has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{3}\right)$

(C) Has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

(D) Have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

82. Let λ be a real number for which the system of linear equations $x + y + z = 6$, $4x + \lambda y - \lambda z = \lambda - 2$ and $3x + 2y - 4z = -5$ has infinitely many solutions. Then λ is a root of the quadratic equation:

(A) $\lambda^2 - 3\lambda - 4 = 0$ **(B)** $\lambda^2 + 3\lambda - 4 = 0$ **(C)** $\lambda^2 - \lambda - 6 = 0$ **(D)** $\lambda^2 + \lambda - 6 = 0$

83. If the system of linear equations $x + y + z = 5$, $x + 2y + 2z = 6$, $x + 3y + \lambda z = \mu, (\lambda, \mu \in R)$, has infinitely many solutions, then the value of $\lambda + \mu$ is:

(A) 7 **(B)** 12 **(C)** 10 **(D)** 9

84. If the system of equations $2x + 3y - z = 0$, $x + ky - 2z = 0$ and $2x - y + z = 0$ has a non-trivial solution (x, y, z), then $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k$ is equal to:

(A) -4 **(B)** $\frac{1}{2}$ **(C)** $-\frac{1}{4}$ **(D)** $\frac{3}{4}$

85. If the system of linear equations $x - 2y + kz = 1$, $2x + y + z = 2$, $3x - y - kz = 3$ has a solution (x, y, z), $z \neq 0$, then (x, y) lies on the straight line whose equation is:

(A) $3x - 4y - 4 = 0$ **(B)** $3x - 4y - 1 = 0$ **(C)** $4x - 3y - 4 = 0$ **(D)** $4x - 3y - 1 = 0$

86. The greatest value of $c \in R$ for which the system of linear equations $x - cy - cz = 0$, $cx - y + cz = 0$, $cx + cy - z = 0$ has a non-trivial solution, is:

(A) -1 **(B)** $1/2$ **(C)** 2 **(D)** 9

87. The set of all values of λ for which the system of linear equation $x - 2y - 2z = \lambda x$, $x + 2y + z = \lambda y$ and $-x - y = \lambda z$ has a non-trivial solution.

(A) Contains exactly two elements **(B)** Contains more than two elements
(C) Is a singleton **(D)** Is an empty set

88. An ordered pair (α, β) for which the system of linear equations

$$(1+\alpha)x + \beta y + z = 2$$
$$\alpha x + (1+\beta)y + z = 3$$
$$\alpha x + \beta y + 2z = 2$$

Has a unique solution, is

(A) (2, 4) **(B)** (–4, 2) **(C)** (1, –3) **(D)** (–3, 1)

89. If the system of linear equations

$$2x + 2y + 3z = a$$
$$3x - y + 5z = b$$
$$x - 3y + 2z = c$$

where a, b, c are non-zero real numbers, has more than one solution, then

(A) $b - c - a = 0$ **(B)** $a + b + c = 0$ **(C)** $b - c + a = 0$ **(D)** $b + c - a = 0$

90. The number of values of $\theta \in (0, \pi)$ for which the system of linear equations

$$x + 3y + 7y = 0$$
$$-x + 4y + 7z = 0$$
$$(\sin 3\theta)x + (\cos 2\theta)y + 2z = 0$$

has a non-trivial solution, is:

(A) Two **(B)** Three **(C)** Four **(D)** One

91. If the system of linear equations, $x - 4y + 7z = g$, $3y - 5z = h$, $-2x + 5y - 9z = k$, is consistent, then:

(A) $2g + h + k = 0$ **(B)** $g + 2h + k = 0$ **(C)** $g + h + k = 0$ **(D)** $g + h + 2k = 0$

92. The system of linear equations, $x + y + z = 2$, $2x + 3y + 2z = 5$, $2x + 3y + (a^2 - 1)z = a + 1$.

(A) Has infinitely many solution for $a = 4$ **(B)** Is inconsistent when $a = 4$
(C) Has a unique solution for $|a| = \sqrt{3}$ **(D)** Is inconsistent when $|a| = \sqrt{3}$

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93. The following system of linear equations **[2020]**

$7x+6y-2z=0$

$3x+4y+2z=0$

$x-2y-6z=0$, has

(A) infinitely many solutions, (x, y, z) satisfying $x=2z$

(B) infinitely many solutions, (x, y, z) satisfying $y=2z$

(C) no solution

(D) only the trivial solution

94. Let $A=[a_{ij}]$ and $B=[b_{ij}]$ be two 3 × 3 matrices such that $b_{ij}=(3)^{(i+j-2)}a_{ji}$, where $i, j=1,2,3.$ If the determinant of B is 81 then the determinant of A is: **[2020]**

(A) 3 **(B)** 1/3 **(C)** 1/81 **(D)** 1/9

95. If the system of linear equations,

$x+y+z=6$

$x+2y+3z=10$

$3x+2y+\lambda z=\mu$

has more than two solutions, then $\mu-\lambda^2$ is equal to _____. **[2020]**

96. If $A=\begin{pmatrix}2 & 2\\ 9 & 4\end{pmatrix}$ and $I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, then $10A^{-1}$ is equal to: **[2020]**

(A) $6I-A$ **(B)** $A-4I$ **(C)** $4I-A$ **(D)** $A-6I$

97. Let $a-2b+c=1$

If $f(x)=\begin{vmatrix}x+a & x+2 & x+1\\ x+b & x+3 & x+2\\ x+c & x+4 & x+3\end{vmatrix}$ then:

(A) $f(50)=-501$ **(B)** $f(-50)=501$ **(C)** $f(-50)=-1$ **(D)** $f(50)=1$ **[2020]**

98. Let α be a root of the equation $x^2+x+1=0$ and the matrix $A=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\ 1 & \alpha & \alpha^2\\ 1 & \alpha^2 & \alpha^4\end{bmatrix}$, then the matrix A^{31} is equal to: **[2020]**

(A) A **(B)** A^3 **(C)** I_3 **(D)** A^2

99. If the system of linear equations,

$2x+2ay+az=0$

$2x+3by+bz=0$

$2x+4cy+cz=0$,

Where $a,b,c\in R$ are non-zero and distinct; has a non-zero solution, then: **[2020]**

(A) $a+b+c=0$ **(B)** a, b, c are in G.P.

(C) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. **(D)** a, b, c are in A. P.

100. For which of the following ordered pairs (μ, δ), the system of linear equations

$x + 2y + 3z = 1$
$3x + 4y + 5z = \mu$
$4x + 4y + 4z = \delta$

is inconsistent ? **[2020]**

(A) (1, 0) **(B)** (4, 3) **(C)** (4, 6) **(D)** (3, 4)

101. The number of all 3×3 matrices A, with entries from the set {–1, 0, 1} such that the sum of the diagonal elements of AA^T is 3, is _____.

102. The system of linear equations $\lambda x + 2y + 2z = 5$, $2\lambda x + 3y + 5z = 8$, $4x + \lambda y + 6z = 10$ has: **[2020]**

(A) No solution when $\lambda = 2$ **(B)** Infinitely many solutions when $\lambda = 2$
(C) No solution when $\lambda = 8$ **(D)** A unique solution when $\lambda = -8$

103. If the matrices $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, $B = adjA$ and $C = 3A$, then $\frac{|adj\, B|}{|C|}$ is equal to : **[2020]**

(A) 72 **(B)** 8 **(C)** 16 **(D)** 2

104. Let S be the set of all $\lambda \in R$ for which the system of linear equations
$2x - y + 2z = 2$; $x - 2y + \lambda z = -4$; $x + \lambda y + z = 4$ has no solution. Then the set S: **[2020]**

(A) Is an empty set **(B)** Contains more than two elements
(C) Is a singleton **(D)** Contains exactly two elements

105. Let A be a 2×2 real matrix with entries from {0, 1} and $|A| \neq 0$. Consider the following two statements:

(P) If $A \neq I_2$, then $|A| = -1$

(Q) If $|A| = 1$, then $tr(A) = 2$, where I_2 denotes 2×2 identity matrix and $tr(A)$ denotes the sum of the diagonal entries of A. Then:

(A) (P) is false and (Q) is true **(B)** (P) is true and (Q) is false **[2020]**
(C) Both (P) and (Q) are false **(D)** Both (P) and (Q) are true

106. Let A be a 3×3 matrix such that $adj\ A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$ and $B = adj\ (adj\ A)$. If $|A| = \lambda$ and $|(B^{-1})^T| = \mu$, then the ordered pair, $(|\lambda|, \mu)$ is equal to: **[2020]**

(A) $\left(9, \frac{1}{81}\right)$ **(B)** $\left(3, \frac{1}{81}\right)$ **(C)** (3, 81) **(D)** $\left(9, \frac{1}{9}\right)$

107. Let S be the set of all integer solutions, (x, y, z), of the system of equations

$x - 2y + 5z = 0$
$-2x + 4y + z = 0$
$-7x + 14y + 9z = 0$

Such that $15 \leq x^2 + y^2 + z^2 \leq 150$. Then the number of elements in the set S is equal to _____ . **[2020]**

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108. If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} =$

$Ax^3 + Bx^2 + Cx + D$, then $B + C$ is equal to: **[2020]**

(A) 1 **(B)** –1 **(C)** –3 **(D)** 9

109. let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}, x \in R$ and $A^4 = [a_{ij}]$. If $a_{11} = 109$, then a_{22} is equal to _____ . **[2020]**

110. If $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}, \left(\theta = \frac{\pi}{24}\right)$ and $A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $i = \sqrt{-1}$, then which one of the following is not true? **[2020]**

(A) $a^2 - b^2 = \frac{1}{2}$ **(B)** $0 \le a^2 + b^2 \le 1$ **(C)** $a^2 - d^2 = 0$ **(D)** $a^2 - c^2 = 1$

111. If the system of equations

$x - 2y + 3z = 9$

$2x + y + z = b$

$x - 7y + az = 24$, has infinitely many solutions, then $a - b$ is equal to ______. **[2020]**

112. If the system of equations

$x + y + z = 2$

$2x + 4y - z = 6$

$3x + 2y + \lambda z = \mu$

Has infinitely many solutions, then: **[2020]**

(A) $2\lambda - \mu = 5$ **(B)** $\lambda - 2\mu = -5$ **(C)** $2\lambda + \mu = 14$ **(D)** $\lambda + 2\mu = 14$

113. Suppose the vector x_1, x_2 and x_3 are the solutions of the system of linear equations, Ax = b when the vector b on the right side is equal to b_1, b_2 and b_3 respectively. If

$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, x_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, b_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$ and $b_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$ then the determinant of A is equal to : **[2020]**

(A) 4 **(B)** $\frac{3}{2}$ **(C)** $\frac{1}{2}$ **(D)** 2

114. Let $\lambda \in R$. The system of linear equations $2x_1 - 4x_2 + \lambda x_3 = 1$, $x_1 - 6x_2 + x_3 = 2$, $\lambda x_1 - 10x_2 + 4x_3 = 3$ is inconsistent for : **[2020]**

(A) exactly two values of λ **(B)** every value of λ
(C) exactly one negative value of λ **(D)** exactly one positive value of λ

115. If the system of linear equations

$x + y + 3z = 0$

$x + 3y + k^2 z = 0$

$3x + y + 3z = 0$

has a non-zero solution (x, y, z) for some $k \in R$, then $x + \left(\frac{y}{z}\right)$ is equal to : **[2020]**

(A) –9 **(B)** 3 **(C)** –3 **(D)** 9

116. If $a + x = b + y = c + z + 1$, where a, b, c, x, y, z are non-zero distinct real numbers, then $\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$ is equal to:

(A) 0 **(B)** $y(a-c)$ **(C)** $y(b-a)$ **(D)** $y(a-b)$ **[2020]**

117. Let m and M be respectively the minimum and maximum values of

$$\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}.$$

Then the ordered pair (m, M) is equal to: **[2020]**

(A) $(-3, 3)$ **(B)** $(-4, -1)$ **(C)** $(-3, -1)$ **(D)** $(1, 3)$

118. The values of λ and μ for which the system of linear equations

$x + y + z = 2$

$x + 2y + 3z = 5$

$x + 3y + \lambda z = \mu$

has infinitely many solution are, respectively: **[2020]**

(A) 5 and 7 **(B)** 5 and 8 **(C)** 6 and 8 **(D)** 4 and 9

119. Let $\theta = \frac{\pi}{5}$ and $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$. If $B = A + A^4$, then $\det(B)$: **[2020]**

(A) lies in $(1, 2)$ **(B)** is zero **(C)** lies in $(2, 3)$ **(D)** is one

120. The sum of distinct values of λ for which the system of equations

$(\lambda-1)x + (3\lambda+1)y + 2\lambda z = 0$

$(\lambda-1)x + (4\lambda-2)y + (\lambda+3)z = 0$

$2x + (3\lambda+1)y + 3(\lambda-1)z = 0,$

has non-zero solutions, is________. **[2020]**

121. Let $a, b, c \in R$ be all non-zero and satisfy $a^3 + b^3 + c^3 = 2$. If the matrix

$$A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$$

Satisfies $A^T A = I$, then a value of abc can be: **[2020]**

(A) $\frac{1}{3}$ **(B)** 3 **(C)** $-\frac{1}{3}$ **(D)** $\frac{2}{3}$

122. Let $A = \left\{ X = (x, y, z)^T : PX = 0 \text{ and } x^2 + y^2 + z^2 = 1 \right\}$, where $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}$ then the set A: **[2020]**

(A) is an empty set **(B)** contains more than two elements

(C) contains exactly two elements **(D)** is a singleton

Matrices and Determinants

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1. If A and B are square matrices of equal degree, then which one is correct among the following? **(1995)**

(A) $A+B=B+A$ **(B)** $A+B=A-B$ **(C)** $A-B=B-A$ **(D)** $AB=BA$

2. If $A=\begin{bmatrix}1&0&0\\0&1&1\\0&-2&4\end{bmatrix}$, $6A^{-1}=A^2+cA+dI$, then (c,d) is: **(2005)**

(A) (–6, 11) **(B)** (–11, 6) **(C)** (11, 6) **(D)** (6, 11)

3. If $P=\begin{bmatrix}\sqrt{3}/2 & 1/2\\ -1/2 & \sqrt{3}/2\end{bmatrix}$, $A=\begin{bmatrix}1&1\\0&1\end{bmatrix}$ and $Q=PAP^T$ then $P^TQ^{2005}P$ is: **(2005)**

(A) $\begin{bmatrix}1&2005\\0&1\end{bmatrix}$ **(B)** $\begin{bmatrix}1&2005\\2005&1\end{bmatrix}$ **(C)** $\begin{bmatrix}1&0\\2005&1\end{bmatrix}$ **(D)** $\begin{bmatrix}1&0\\0&1\end{bmatrix}$

4. If $A=\begin{bmatrix}\alpha&0\\1&1\end{bmatrix}$ and $B=\begin{bmatrix}1&0\\5&1\end{bmatrix}$, then value of α for which $A^2=B$, is: **(2003)**

(A) 1 **(B)** –1 **(C)** 4 **(D)** No real values

5. Let $P=[a_{ij}]$ be a 3×3 matrix and let $Q=[b_{ij}]$, where $b_{ij}=2^{i+j}a_{ij}$ for $1\le i,\ j\le 3$. If the determinant of P is 2, then the determinant of the matrix Q is: **[2012]**

(A) 2^{10} **(B)** 2^{11} **(C)** 2^{12} **(D)** 2^{13}

6. If $A=\begin{bmatrix}\alpha&2\\2&\alpha\end{bmatrix}$ and $|A^3|=125$, then the value of α is: **(2004)**

(A) ±1 **(B)** ±2 **(C)** ±3 **(D)** ±5

7. The number of distinct real roots of $\begin{vmatrix}\sin x&\cos x&\cos x\\\cos x&\sin x&\cos x\\\cos x&\cos x&\sin x\end{vmatrix}=0$ in the interval $-\frac{\pi}{4}\le x\le\frac{\pi}{4}$ is: **(2001)**

(A) 0 **(B)** 2 **(C)** 1 **(D)** 3

8. If $f(x)=\begin{vmatrix}1&x&x+1\\2x&x(x-1)&(x+1)x\\3x(x-1)&x(x-1)(x-1)&(x+1)x(x-1)\end{vmatrix}$, then $f(100)$ is equal to: **(1999)**

(A) 0 **(B)** 1 **(C)** 100 **(D)** –100

9. The parameter on which the value of the determinant $\begin{vmatrix}1&a&a^2\\\cos(p-d)x&\cos px&\cos(p+d)x\\\sin(p-d)x&\sin px&\sin(p+d)x\end{vmatrix}$ does not depend upon, is: **(1997)**

(A) a **(B)** p **(C)** d **(D)** x

10. The determinant $\begin{vmatrix}xp+y&x&y\\yp+z&y&z\\0&xp+y&yp+z\end{vmatrix}=0$, if **(1997)**

(A) x,y,z are in AP **(B)** x,y,z are in GP **(C)** x,y,z are in HP **(D)** xy,yz,zx are in AP

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11. Consider the set a of all determinants of order 3 with entries 0 or 1 only. Let B be the subset of A consisting of all determinants with value 1. Let C be the subset of A consisting of all determinants with value –1. Then: **(1981)**

(A) C is empty **(B)** B has as many elements as C

(C) $A = B \cup C$ **(D)** B has twice as many elements as C

12. If A is 3×3 non-singular matrix such that $AA^T = A^T A$ and $B = A^{-1}A^T$, then BB^T is equal to: **(2014)**

(A) $I + B$ **(B)** I **(C)** B^{-1} **(D)** $(B^{-1})^T$

13. If P is a 3×3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3×3 identity matrix, then there exists a column matrix, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that **(2012)**

(A) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ **(B)** $PX = X$ **(C)** $PX = 2X$ **(D)** $PX = -X$

14. Let $\omega \neq 1$ be a cube root of unity and s be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$,

where each of a, b and c is either ω or ω^2, Then, the number of distinct matrices in the set S is: **(2011)**

(A) 2 **(B)** 6 **(C)** 4 **(D)** 8

15. Let M and N be two 3×3 non-singular skew-symmetric matrices such that $MN = NM$. If P^T denotes the transpose of P, then $M^2N^2(M^TN)^{-1}(MN^{-1})^T$ is equal to: **(2011)**

(A) M^2 **(B)** $-N^2$ **(C)** $-M^2$ **(D)** MN

16. The number of 3×3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions, is: **(2010)**

(A) 0 **(B)** $2^9 - 1$ **(C)** 168 **(D)** 2

17. given, $2x - y + 2z = 2$, $x - 2y + z = -4$, $x + y + \lambda z = 4$, then the value of λ such that the given system of equations has no solution, is: **(2004)**

(A) 3 **(B)** 1 **(C)** 0 **(D)** –3

18. The number of values of k for which the system of equations $(k+1)x+8y=4k$ and $kx+(k+3)y=3k-1$ has infinitely many solutions, is/are: **(2002)**

(A) 0 **(B)** 1 **(C)** 2 **(D)** ∞

19. If the system of equations $x-ky-z=0$, $kx-y-z=0$, $x+y-z=0$ has a non-zero solution, then possible values of k are: **(2000)**

(A) –1, 2 **(B)** 1, 2 **(C)** 0, 1 **(D)** –1, 1

Assertion and Reason

For the following questions, choose the correct answer from the codes (a), (b), (c) and (d) defined as follows:

(A) Statement I is true, Statement II is also true; Statement II is the correct explanation of Statement I

(B) Statement I is true, Statement II is also true; Statement II is not the correct explanation of Statement I

(C) Statement I is true; Statement II is false.

(D) Statement I is false; Statement II is true.

20. Consider the system of equation $x-2y+3z=-1$, $x-3y+4z=1$ and $-x+y-2z=k$

Statement I : The system of equations has no solution for $k \neq 3$ and

Statement II : The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$ for $k \neq 0$. **(2008)**

21. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3. If $Q = \left[q_{ij}\right]$ is a matrix such that $P^{50} - Q = I$, then $\dfrac{q_{31}+q_{32}}{q_{21}}$ equals :

(A) 52 **(B)** 103 **(C)** 201 **(D)** 205

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22. How many 3×3 matrices M with entries from $\{0,1,2\}$ are there, for which the sum of the diagonal entries of M^TM is 5 ?

(A) 126 **(B)** 198 **(C)** 162 **(D)** 135

***23.** For 3×3 matrices M and N, which of the following statement(s) is/are not correct? **(2013)**

(A) N^TMN is symmetric or skew-symmetric, according as M is symmetric or skew-symmetric

(B) $MN-NM$ is symmetric for all symmetric matrices M and N

(C) MN is symmetric for all symmetric matrices M and N

(D) $(adj\,M)(adj\,N)=adj(MN)$ for all invertible matrices M and N

***24.** Let M and N be two 3×3 matrices such that $MN=NM$. Further, if $M\neq N^2$ and $M^2=N^4$, then:

(A) determinant of (M^2+MN^2) is 0 **(2014)**

(B) there is a 3×3 non-zero matrix U such that $(M^2+MN^2)\,U$ is zero matrix

(C) determinant of $(M^2+MN^2)\geq1$

(D) for a 3×3 matrix U, if $(M^2+MN^2)U$ equals the zero matrix, then U is the zero matrix

***25.** Let ω be a complex cube root of unity with $\omega\neq0$ and $P=[p_{ij}]$ be an $n\times n$ matrix with $p_{ij}=\omega^{i+j}$. Then $p^2\neq0$ when n is equal to: **(2013)**

(A) 57 **(B)** 55 **(C)** 58 **(D)** 56

***26.** The determinant $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}$ is equal to zero, then: **(1986)**

(A) a,b,c are in AP **(B)** a,b,c are in GP

(C) a,b,c are in HP **(D)** $(x-\alpha)$ is a factor of $ax^2+2bx+c$

***27.** Let M be a 2×2 symmetric matrix with integer entries. Then, M is invertible, if: **(2014)**

(A) the first column of M is the transpose of the second row of M

(B) the second row of M is the transpose of the first column of M

(C) M is a diagonal matrix with non-zero entries in the main diagonal

(D) the product of entries in the main diagonal of M is not the square of an integer

***28.** If the adjoint of a 3×3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of P is/are: **(2014)**

(A) –2 **(B)** –1 **(C)** 1 **(D)** 2

***29.** Let X and Y be two arbitrary, 3×3, non-zero, skew – symmetric matrices and Z be an arbitrary 3×3, non-zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric?

(A) $Y^3Z^4-Z^4Y^3$ **(B)** $X^{44}+Y^{44}$ **(C)** $X^4Z^3-Z^3X^4$ **(D)** $X^{23}+Y^{23}$

***30.** Which of the following vales of α satisfy the equation $\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix}=-648\,\alpha$?

(A) –4 **(B)** 9 **(C)** –9 **(D)** 4

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***31.** Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in \mathbb{R}$. Suppose $Q = \left[q_{ij}\right]$ is a matrix such that $PQ = kI$, where $k \in \mathbb{R}, k \neq 0$ and I the identity matrix of order 3. If $q_{23} = \frac{-k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then :

(A) $\alpha = 0, k = 8$
(B) $4\alpha - k + 8 = 0$
(C) $\det\left(P\ adj\left(Q\right)\right) = 2^9$
(D) $\det\left(Q\ adj\left(P\right)\right) = 2^{13}$

***32.** Let $a, \lambda, \mu \in R$. Consider the system of linear equations $ax + 2y = \lambda$, $3x - 2y = \mu$
Which of the following statement(s) is(are) correct ?

(A) If $a = -3$, then the system has infinitely many solutions for all values of λ and μ
(B) If $a \neq -3$, then the system has a unique solution for all values of λ and μ
(C) If $\lambda + \mu = 0$, then the system has infinitely many solutions for $a = -3$
(D) If $\lambda + \mu \neq 0$, then the system has no solution for $a = -3$

***33.** Which of the following is (are) NOT the square of a 3 × 3 matrix with real entries ?

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$
(B) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$
(C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
(D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

***34.** Let S be the set of all column matrices $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $b_1, b_2, b_3 \in \mathbb{R}$ and the system of equations (in real variables) $-x + 2y + 5z = b_1$, $2x - 4y + 3z = b_2$, $x - 2y + 2z = b_3$
has at least one solution. Then, which of the following system(s) (in real variables) has(have) at least one solution for each $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in S$?

(A) $x + 2y + 3z = b_1$, $4y + 5z = b_2$ and $x + 2y + 6z = b_3$
(B) $x + y + 3z = b_1$, $5x + 2y + 6z = b_2$ and $-2x - y - 3z = b_3$
(C) $-x + 2y - 5z = b_1$, $2x - 4y + 10z = b_2$ and $x - 2y + 5z = b_3$
(D) $x + 2y + 5z = b_1$, $2x + 3z = b_2$ and $x + 4y - 5z = b_3$

35. Let $M = \begin{bmatrix} \sin^4\theta & -1 - \sin^2\theta \\ 1 + \cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$, where $\alpha = \alpha(\theta)$ and $\beta = \beta(\theta)$ are real numbers, and I is the 2×2 identity matrix. If α^* is the minimum of the set $\{\alpha(\theta) : \theta \in [0, 2\pi)\}$ and β^* is the minimum of the set $\{\beta(\theta) : \theta \in [0, 2\pi)\}$. Then the value of $\alpha^* + \beta^*$ is :

(A) $-\frac{37}{16}$
(B) $-\frac{31}{16}$
(C) $-\frac{29}{16}$
(D) $-\frac{17}{16}$

***36.** Let $M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix}$ and $adj\,M = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$

Where a and b are real numbers. Which of the following options is/are correct?

(A) If $M\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then $\alpha - \beta + \gamma = 3$ **(B)** $(adj\,M)^{-1} + adjM^{-1} = -M$

(C) $a + b = 3$ **(D)** $\det(adj\,M^2) = 81$

***37.** Let $P_1 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $P_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$P_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, $P_5 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $P_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ and $X = \sum_{k=1}^{6} P_k \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} P_k^T$

where P_k^T denotes the transpose of the matrix P_k. Then which of the following options is(are) correct ?

(A) The sum of diagonal entries of X is 18 **(B)** X is a symmetric matrix

(C) $X - 30I$ is an invertible matrix **(D)** If $X\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \alpha\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, then $\alpha = 30$

***38.** Let $x \in \mathbb{R}$ and let $P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$, $Q = \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix}$ and $R = PQP^{-1}$.

Then which of the following options is(are) correct ?

(A) For $x = 0$, if $R\begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6\begin{bmatrix} 1 \\ a \\ b \end{bmatrix}$, then $a + b = 5$

(B) $\det R = \det\begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8$, for all $x \in \mathbb{R}$

(C) For $x = 1$, there exists a unit vector $\alpha\hat{i} + \beta\hat{i} + \gamma\hat{k}$ for which $R\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(D) There exists a real number x such that PQ = QP

39. For positive numbers x, y and z, the numerical value of the determinant $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is......

(1993)

40. The value of the determinant $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix}$ is...... **(1988)**

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41. Given that $x = -9$ is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$, the other two roots are ...... and .... **(1983)**

42. The solution set of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ is ...... **(1981)**

43. Let $p\lambda^4 + q\lambda^3 + r\lambda^2 + s\lambda + t = \begin{vmatrix} \lambda^2+3\lambda & \lambda-1 & \lambda+3 \\ \lambda+1 & -2\lambda & \lambda-4 \\ \lambda-3 & \lambda+4 & 3\lambda \end{vmatrix}$ be an identity in λ, where p, q, r, s and t are constants. Then, the value of t is ...... **(1981)**

44. Let k be a positive real number and let $A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$

If $\det(adjA) + \det(adjB) = 10^6)$, then $[k]$ is equal to .... **(2010)**

45. The system of equations $\lambda x + y + z = 0$, $-x + \lambda y + z = 0$ and $-x - y + \lambda z = 0$ will have a non-zero solution, if real values of λ are given by... **(1982)**

46. Let M be a 3×3 matrix satisfying $M\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$, $M\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, and $M\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$, then the sum of the diagonal entries of M is .... **(2011)**

47. Let $z = \dfrac{-1+\sqrt{3}\,i}{2}$, where $i = \sqrt{-1}$, and $r, s \in \{1, 2, 3\}$. Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and I be the identity matrix of order 2. Then the total number of ordered pairs (r, s) for which $P^2 = -I$ is _________.

48. The total number of distinct $x \in R$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is _________.

49. For a real number α, if the system $\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ of linear equations, has infinitely many solutions, then $1 + \alpha + \alpha^2 =$

50. Let P be a matrix of order 3×3 such that all the entries in P are from the set $\{-1, 0, 1\}$. Then, the maximum possible value of the determinant of P is _____ .

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Paragraph for Questions 51 – 53 **(2011)**

Let a, b and c be three real numbers satisfying $[a\ b\ c]\begin{vmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{vmatrix} = [0\ 0\ 0]$ ....(i)

51. If the point $P(a, b, c)$, with reference to Eq.(i), lies on the plane $2x + y + z = 1$, then the value of $7a+b+c$ is:

(A) 0 **(B)** 12 **(C)** 7 **(D)** 6

52. Let $b = 6$, with a and c satisfying Eq.(i). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\sum_{n=0}^{\infty}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^n$ is equal to:

(A) 6 **(B)** 7 **(C)** 6/7 **(D)** ∞

53. Let ω be a solution of $x^3 - 1 = 0$ with $\text{Im}(\omega) > 0$. If $a = 2$ with b and c satisfying Eq.(i) then the value of $\frac{3}{\omega^4} + \frac{1}{\omega^b} + \frac{1}{\omega^c}$ is:

(A) –2 **(B)** 2 **(C)** 3 **(D)** –3

Paragraph for Questions 54 – 56 **(2010)**

Let p be an odd prime number and T_p be the set of 2×2 matrices $T_p = \left\{A = \begin{bmatrix} a & b \\ c & a \end{bmatrix}\right\}; a, b, c \in \{0, 1, 2, \ldots, p-1\}$

54. The number of A in T_p such that $\det(A)$ is not divisible by p, is:

(A) $2p^2$ **(B)** $p^3 - 5p$ **(C)** $p^3 - 3p$ **(D)** $p^3 - p^2$

55. The number of A in T_p such that the trace of A is not divisible by p but $\det(A)$ is divisible by p is:

(A) $(p-1)(p^2-p+1)$ **(B)** $p^3 - (p-1)^2$ **(C)** $(p-1)^2$ **(D)** $(p-1)(p^2 - 2)$

56. The number of A in T_p such that A is either symmetric or skew-symmetric or both and $\det(A)$ is divisible by p is: [Note: the trace of a matrix is the sum of its diagonal entries.]

(A) $(p-1)^2$ **(B)** $2(p-1)$ **(C)** $(p-1)^2 + 1$ **(D)** $2p - 1$

57. If M is a 3×3 matrix, where $M^T M = I$ and $\det(M) = 1$, then prove that $\det(M - 1) = 0$ **(2004)**

58. If matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, where a, b, c are real positive numbers, $abc = 1$ and $A^T A = I$, then find the value of $a^3 + b^3 + c^3$. **(2003)**

59. Let a, b, c be real numbers with $a^2 + b^2 + c^2 = 1$. Show that the equation

$$\begin{vmatrix} ax - by - c & bx + ay & cx + a \\ bx + ay & -ax + by - c & cy + b \\ cx + a & cy + b & -ax - by + c \end{vmatrix} = 0$$ represents a straight line. **(2001)**

60. Prove that for all value of θ $\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta + \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(2\theta + \frac{4\pi}{3}\right) \\ \sin\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(2\theta - \frac{4\pi}{3}\right) \end{vmatrix} = 0$ **(2000)**

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61. Suppose, $f(x)$ is a function satisfying the following conditions: **(1998)**

(a) $f(0)=2,\ f(1)=1$

(b) f has a minimum value at $x=\frac{5}{2}$, and

(c) for all x, $f'(x)=\begin{vmatrix} 2ax & 2ax-1 & 2ax+b+1 \\ b & b+1 & -1 \\ 2(ax+b) & 2ax+2b+1 & 2ax+b \end{vmatrix}$

where a, b are some constants. Determine the constants a, b and the function $f(x)$.

62. Find the value of the determinant $\begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$, where a, b and c are respectively the p^{th}, q^{th} and r^{th} terms of a harmonic progression. **(1997)**

63. Let $a>0, d>0$. Find the value of the determinant

$$\begin{vmatrix} \frac{1}{a} & \frac{1}{a(a+d)} & \frac{1}{(a+d)(a+2d)} \\ \frac{1}{(a+d)} & \frac{1}{(a+d)(a+2d)} & \frac{1}{(a+2d)(a+3d)} \\ \frac{1}{(a+2d)} & \frac{1}{(a+2d)(a+3d)} & \frac{1}{(a+3d)(a+4d)} \end{vmatrix}$$

(1997)

64. For all values of A, B, C and P, Q, R, show that **(1994)**

$$\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix}=0$$

65. For fixed positive integer n, if $D=\begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$ then show that $\left[\frac{D}{(n!)^3}-4\right]$ is divisible by n.

(1992)

66. If $a\neq p, b\neq q, c\neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix}=0$. Then, find the value of $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}$ **(1991)**

67. Let the three digit numbers A28, 3B9 and 62C, where A, B and C are integers between 0 and 9, be divisible by a fixed integer k. Show that the determinant $\begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$ is divisible by k. **(1990)**

68. Let $\Delta_a = \begin{vmatrix} a-1 & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$. Show that $\sum_{a=1}^{n} \Delta_a = c \in$ constant. **(1989)**

69. Show that $\begin{vmatrix} {}^{x}C_r & {}^{x}C_{r+1} & {}^{x}C_{r+2} \\ {}^{y}C_r & {}^{y}C_{r+1} & {}^{y}C_{r+2} \\ {}^{z}C_r & {}^{z}C_{r+1} & {}^{z}C_{r+2} \end{vmatrix} = \begin{vmatrix} {}^{x}C_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^{y}C_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^{z}C_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix}$ **(1985)**

70. If α be a repeated root of a quadratic equation $f(x) = 0$ and $A(x), B(x)$ and $C(x)$ be polynomials of degree 3, 4 and 5 respectively, then show that:

$\begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$ is divisible by $f(x)$, where prime denotes the derivatives. **(1984)**

71. Without expanding a determinant at any stage, show that: $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = xA + B$

Where A and B are determinants of order 3 not involving x. **(1982)**

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72. Let a, b, c be positive and not all equal. Show that the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative. **(1981)**

73. $A = \begin{bmatrix} a & 0 & 1 \\ 1 & c & b \\ 1 & d & b \end{bmatrix}, B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}, U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}, V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$

If there is a vector matrix X, such that $AX = U$ has infinitely many solutions, then prove that $BX = V$ cannot have a unique solution. If $a\ f\ d \neq 0$. Then, prove that $BX = V$ has no solution. **(2004)**

74. Let λ and α be real. Find the set of all values of λ for which the system of linear equations

$\lambda x + (\sin\alpha)y + (\cos\alpha)z = 0$

$x + (\cos\alpha)y + (\sin\alpha)z = 0$ and $-x + (\sin\alpha)y - (\cos\alpha)z = 0$

has a non-trivial solution. For $\lambda = 1$, find all values of α. **(1993)**

75. Let $\alpha_1, \alpha_2, \beta_1, \beta_2$ be the roots of $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ respectively. If the system of equations $\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y + \beta_2 z = 0$ has a non-trivial solution, then prove that $\frac{b^2}{q^2} = \frac{ac}{pr}$. **(1987)**

76. Consider the system of linear equations in x, y, z

$(\sin\theta)x - y + z = 0$, $(\cos 2\theta)x + 4y + 3z = 0$ and $2x + 7y + 7z = 0$

Find the values of θ for which this system has non-trivial solution. **(1986)**

77. Show that the system of equations, $3x - y + 4z = 3$, $x + 2y - 3z = -2$ and $6x + 5y + \lambda z = -3$ has atleast one solution for any real number $\lambda \neq -5$. Find the set of solutions, if $\lambda = -5$. **(1983)**

78. For what values of *m*, does the system of equations $3x + my = m$ and $2x - 5y = 20$ has a solution satisfying the conditions $x > 0, y > 0$? **(1979)**

79. For what value of *k*, does the following system of equations possess a non-trivial solution over the set of rationals $x + y - 2z = 0, 2x - 3y + z = 0$ and $x - 5y + 4z = k$. Find all the solutions. **(1979)**

80. Given, $x = cy + bz, y = az + cx, z = bx + ay$, where x, y, z are not all zero, prove that

$a^2 + b^2 + c^2 + 2ab = 1$ **(1978)**

81. Let M be a 3×3 invertible matrix with real entries and let I denote the 3×3 identity matrix. If $M^{-1} = adj(adj\, M)$, then which of the following statement is/are ALWAYS TRUE? **(2020)**

(A) $M = I$ **(B)** $\det M = I$ **(C)** $M^2 = I$ **(D)** $(adj\, M)^2 = I$

82. The trace of a square matrix is defined to be the sum of its diagonal entries. If A is a 2×2 matrix such that the trace of A is 3 and the trace of A^3 is –18, then the value of the determinant of A is ____. **(2020)**